

Q1.

(a)

We want to simplify:

$$\frac{15a^2b}{7cd^2} \div \frac{5a^3b^2}{21c^2d^3}.$$

1) Rewrite the division as multiplication by the reciprocal

$$\frac{15a^2b}{7cd^2} \div \frac{5a^3b^2}{21c^2d^3} = \frac{15a^2b}{7cd^2} \times \frac{21c^2d^3}{5a^3b^2}.$$

2) Combine into a single fraction

$$= \frac{15a^2b \times 21c^2d^3}{7cd^2 \times 5a^3b^2} = \frac{15 \times 21 \times a^2 \times b \times c^2 \times d^3}{7 \times c \times d^2 \times 5 \times a^3 \times b^2}.$$

3) Multiply numerical coefficients

$$15 \times 21 = 315 \quad \text{and} \quad 7 \times 5 = 35.$$

Hence the numerical part is

$$\frac{315}{35} = 9.$$

4) Combine like terms of variables

$$a^2 \div a^3 = \frac{1}{a}, \quad b^1 \div b^2 = \frac{1}{b}, \quad c^2 \div c^1 = c, \quad d^3 \div d^2 = d.$$

Putting it all together:

$$\frac{315}{35} \times \frac{cd}{ab} = 9 \times \frac{cd}{ab} = \frac{9cd}{ab}.$$

$$\boxed{\frac{9cd}{ab}}$$

(b)

Given:

$$2x^2 - x - 2 = 0$$

1. Identify the coefficients

$$a = 2, \quad b = -1, \quad c = -2$$

2. Apply the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plug in $a = 2$, $b = -1$, and $c = -2$:

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-2)}}{2 \cdot 2} \\ &= \frac{1 \pm \sqrt{1 + 16}}{4} = \frac{1 \pm \sqrt{17}}{4} \end{aligned}$$

3. Simplify the Radicand

$$\sqrt{17} \approx 4.1231 \quad (\text{to 4 decimal places})$$

Therefore, the two solutions become:

$$\begin{aligned} x_1 &= \frac{1 + 4.1231}{4} = \frac{5.1231}{4} \approx 1.2808, \\ x_2 &= \frac{1 - 4.1231}{4} = \frac{-3.1231}{4} \approx -0.7808 \end{aligned}$$

4. Round to Two Decimal Places

$$x_1 \approx 1.28, \quad x_2 \approx -0.78$$

$$\boxed{x \approx 1.28 \quad \text{or} \quad x \approx -0.78}$$

Q2.

(a)

(i): Probability Both Are Blue

1. Probability of selecting a blue ball from Box A

$$P(\text{Blue from A}) = \frac{3}{5}$$

2. Probability of selecting a blue ball from Box B

$$P(\text{Blue from B}) = \frac{2}{6} = \frac{1}{3}$$

3. Because the choices from the two boxes are independent,

$$P(\text{Both blue}) = P(\text{Blue from A}) \times P(\text{Blue from B}) = \frac{3}{5} \times \frac{1}{3} = \frac{3}{15} = \frac{1}{5}$$

(ii): Probability of Different Colours

To be different colours, either:

1. Box A picks Green and Box B picks Blue, or
2. Box A picks Blue and Box B picks Green.

We compute each case and then sum them (since these events are mutually exclusive):

1. $P(A = \text{Green and } B = \text{Blue})$

$$P(A = \text{Green}) = \frac{2}{5}, \quad P(B = \text{Blue}) = \frac{2}{6} = \frac{1}{3}$$

So,

$$P(A = \text{Green and } B = \text{Blue}) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

2. $P(A = \text{Blue and } B = \text{Green})$

$$P(A = \text{Blue}) = \frac{3}{5}, \quad P(B = \text{Green}) = \frac{4}{6} = \frac{2}{3}$$

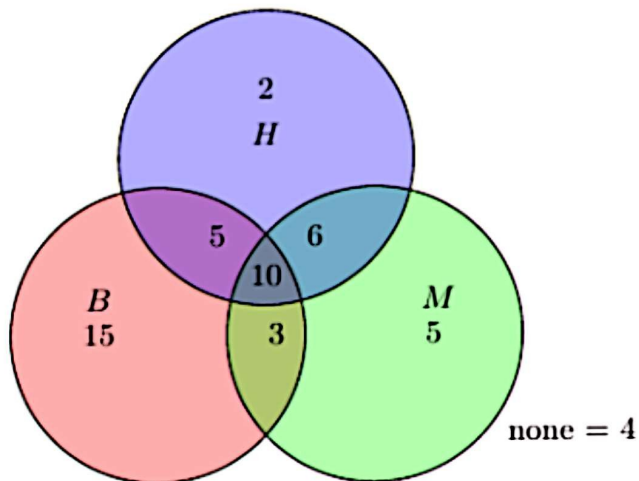
So,

$$P(A = \text{Blue and } B = \text{Green}) = \frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$$

3. Sum these probabilities

$P(\text{Different colours}) = \frac{2}{15} + \frac{6}{15} = \frac{8}{15}$
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(b) (i)



(ii)

(a) How many learners collected none of the textbooks?

Use the principle of inclusion-exclusion to find the number of learners who collected at least one type of textbook:

$$\begin{aligned}n(B \cup M \cup H) &= n(B) + n(M) + n(H) \\&\quad - [n(B \cap M) + n(B \cap H) + n(M \cap H)] \\&\quad + n(B \cap M \cap H) \\&= 33 + 24 + 23 \\&\quad - (13 + 15 + 16) \\&\quad + 10 \\&= 80 - 44 + 10 \\&= 46\end{aligned}$$

Hence, the number who collected none is:

$$50 - 46 = 4$$

(b) How many learners collected exactly one type of textbook?

- **Exactly Biology only:**

$$\begin{aligned}B_{\text{only}} &= n(B) - n(B \cap M) - n(B \cap H) + n(B \cap M \cap H) \\&= 33 - 13 - 15 + 10 = 15\end{aligned}$$

- **Exactly Mathematics only:**

$$\begin{aligned}M_{\text{only}} &= n(M) - n(B \cap M) - n(M \cap H) + n(B \cap M \cap H) \\&= 24 - 13 - 16 + 10 = 5\end{aligned}$$

- **Exactly History only:**

$$\begin{aligned}H_{\text{only}} &= n(H) - n(B \cap H) - n(M \cap H) + n(B \cap M \cap H) \\&= 23 - 15 - 16 + 10 = 2\end{aligned}$$

(c) How many learners collected exactly two types of textbooks?

For each pair intersection, subtract those in the triple intersection (to exclude the triple collectors):

- **Exactly $B \cap M$ (but not H):**

$$n(B \cap M) - n(B \cap M \cap H) = 13 - 10 = 3$$

- **Exactly $B \cap H$ (but not M):**

$$n(B \cap H) - n(B \cap M \cap H) = 15 - 10 = 5$$

- **Exactly $M \cap H$ (but not B):**

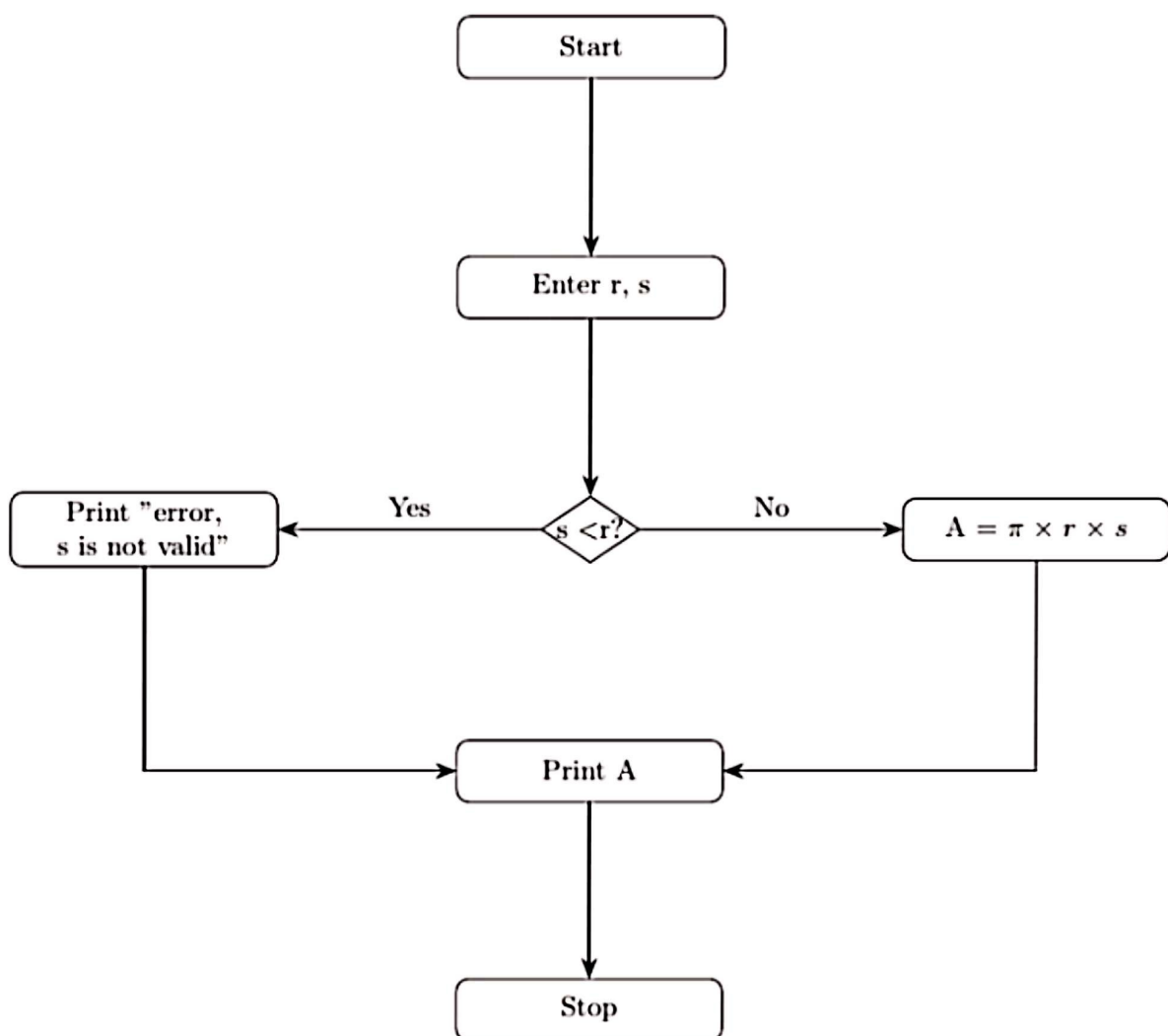
$$n(M \cap H) - n(B \cap M \cap H) = 16 - 10 = 6$$

So, the total who collected **exactly two types**:

$$3 + 5 + 6 = 14$$

Q3.

(a)



(b) (i)

(a) Express FH in terms of a and/or b

To find FH , we can use the vector addition rule in triangle EFH :

$$EF + FH = EH$$

Therefore,

$$FH = EH - EF$$

Substituting the given vectors $EH = 4a$ and $EF = 4b$:

$$FH = 4a - 4b = 4(a - b)$$

(b) Express FG in terms of a and/or b

Since H is the midpoint of EG , we know that $EH = HG$.

Therefore,

$$EG = EH + HG = EH + EH = 2EH$$

Given $EH = 4a$, we have:

$$EG = 2(4a) = 8a$$

Now, we can use the vector addition rule in triangle EFG :

$$EF + FG = EG$$

Therefore,

$$FG = EG - EF$$

Substituting the vectors $EG = 8a$ and $EF = 4b$:

$$FG = 8a - 4b = 4(2a - b)$$

(c) Express EJ in terms of a and/or b

We are given that $FJ : JG = 1 : 3$. This means that point J divides the line segment FG in the ratio $1 : 3$. We can use the section formula to find FJ :

$$FJ = \frac{1}{1+3}FG = \frac{1}{4}FG$$

From part (b), we know $FG = 4(2a - b)$, so:

$$FJ = \frac{1}{4} \times 4(2a - b) = 2a - b$$

Now, we can find EJ using the vector addition rule in triangle EFJ :

$$EF + FJ = EJ$$

Therefore,

$$EJ = EF + FJ$$

Substituting the vectors $EF = 4b$ and $FJ = 2a - b$:

$$EJ = 4b + (2a - b) = 2a + 3b$$

(ii)

From part (i)(a), we already found that:

$$FH = 4(a - b)$$

Given that $FP = kFH$, we can substitute the expression for FH :

$$FP = k[4(a - b)]$$

$$FP = 4k(a - b)$$

$$FP = 4ka - 4kb$$

Now we need to express EP in terms of EF and FP . We can use the vector addition rule in triangle $EF P$:

$$EF + FP = EP$$

So,

$$EP = EF + FP$$

We are given that $EF = 4b$. Substitute this and the expression we found for FP into the equation for EP :

$$EP = 4b + (4ka - 4kb)$$

Rearrange the terms to group the terms with a and b :

$$EP = 4ka + 4b - 4kb$$

Factor out 4 from the terms with b :

$$EP = 4ka + 4(1 - k)b$$

Therefore, we have successfully shown that if $FP = kFH$:

$$\boxed{EP = 4ka + 4(1 - k)b}$$

Q4.

(a)

(i) Determinant of P

Given

$$P = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix},$$

the determinant $\det(P)$ is:

$$\det(P) = 3 \times 2 - (-1) \times (-4) = 6 - 4 = 2$$

(ii) Inverse of P

For a 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

its inverse (if the determinant is nonzero) is:

$$\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Here, $a = 3$, $b = -1$, $c = -4$, $d = 2$, and we already found $\det(P) = 2$. So:

1. Adjoint/“swap and negate” step

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$

2. Divide by $\det(P)$

$$P^{-1} = \frac{1}{2} \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$$

Page 11 of 31

(b)

(i) 10th Term

The n th term of a G.P. is given by $T_n = ar^{n-1}$. Thus,

$$T_{10} = 10 \times 3^{10-1} = 10 \times 3^9 = 10 \times 19,683 = 196,830$$

(ii) Sum of the First 6 Terms

The sum of the first n terms of a G.P. is

$$S_n = a \frac{r^n - 1}{r - 1}.$$

For $n = 6$:

$$S_6 = 10 \times \frac{3^6 - 1}{3 - 1} = 10 \times \frac{729 - 1}{2} = 10 \times \frac{728}{2} = 10 \times 364 = 3,640$$

(iii) Geometric Mean of 2,430 and 21,870

The geometric mean (G.M.) of two positive numbers x and y is $\sqrt{xy}$.
Hence,

$$\text{G.M. of } (2430, 21870) = \sqrt{2430 \times 21870}$$

Factoring each number:

$$2430 = 3^5 \times 10$$

$$21870 = 3^7 \times 10$$

So,

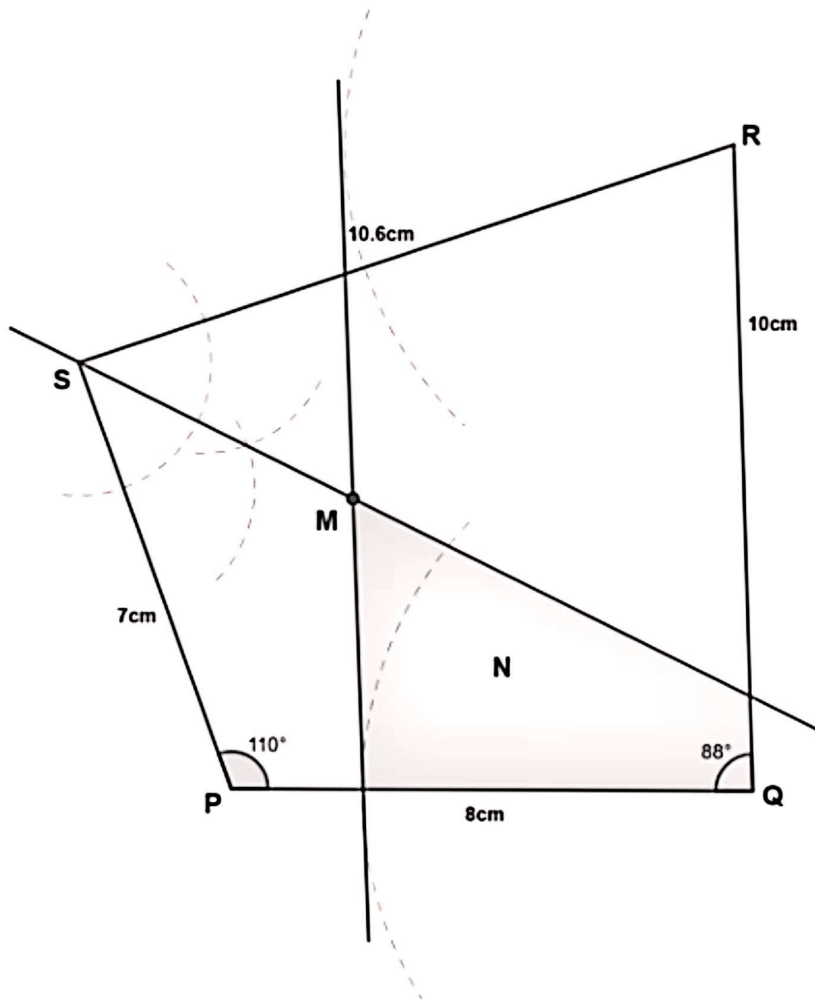
$$2430 \times 21870 = 3^5 \times 10 \times 3^7 \times 10 = 3^{5+7} \times 10 \times 10 = 3^{12} \times 100$$

Taking the square root:

$$\sqrt{3^{12} \times 100} = 3^6 \times 10 = 729 \times 10 = 7,290$$

Thus, the geometric mean is 7,290

Q5.



Q6.

(a)

We want to evaluate the definite integral:

$$\int_1^3 (4x^3 - 3x^2 + 2) dx$$

1: Find the antiderivative

- Integrate $4x^3$:

$$\int 4x^3 dx = 4 \cdot \frac{x^4}{4} = x^4$$

- Integrate $-3x^2$:

$$\int -3x^2 dx = -3 \cdot \frac{x^3}{3} = -x^3$$

- Integrate 2:

$$\int 2 dx = 2x$$

Thus, an antiderivative $F(x)$ is:

$$F(x) = x^4 - x^3 + 2x$$

2: Evaluate at the limits

We need to compute $F(3)$ and $F(1)$.

$$F(3) = 3^4 - 3^3 + 2 \cdot 3 = 81 - 27 + 6 = 60$$

$$F(1) = 1^4 - 1^3 + 2 \cdot 1 = 1 - 1 + 2 = 2$$

3: Subtract

$$\int_1^3 (4x^3 - 3x^2 + 2) dx = F(3) - F(1) = 60 - 2 = 58$$

Hence, the value of the definite integral is:

$$\boxed{58}$$

(b)

We want to combine:

$$\frac{1}{x-1} - \frac{10}{2x+1}$$

into a single fraction in simplest form

1. Identify the common denominator

The denominators are $(x-1)$ and $(2x+1)$. Their product gives the common denominator:

$$(x-1)(2x+1).$$

2. Rewrite each term over the common denominator:

$$\frac{1}{x-1} = \frac{2x+1}{(x-1)(2x+1)}, \quad \frac{10}{2x+1} = \frac{10(x-1)}{(2x+1)(x-1)}$$

3. Combine and simplify the numerator

Combine over the common denominator:

$$\frac{1}{x-1} - \frac{10}{2x+1} = \frac{2x+1}{(x-1)(2x+1)} - \frac{10(x-1)}{(2x+1)(x-1)} = \frac{(2x+1) - 10(x-1)}{(x-1)(2x+1)}$$

Simplify the numerator:

$$(2x+1) - 10(x-1) = 2x+1 - 10x+10 = -8x+11$$

Hence, the expression in simplest form is:

$$\frac{-8x+11}{(x-1)(2x+1)}$$

Q7.

(a)

(i) Finding the perpendicular height MV

Because the pyramid is a right pyramid, the apex V is vertically above M . Therefore, MV is perpendicular to the base

1. Find PM :

The diagonal of rectangle $PQRS$ has length

$$PR = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$

Since M is the midpoint of the rectangle, PM is half of PR :

$$PM = \frac{1}{2} \times PR = 5$$

2. Use right triangle PVM :

In $\triangle PVM$, we know $PV = 13$ cm and $PM = 5$ cm. Using the Pythagorean theorem,

$$MV = \sqrt{PV^2 - PM^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$$

Hence, the perpendicular height of the pyramid is

$$MV = 12 \text{ cm}$$

(ii) Finding the volume of the pyramid

The volume V_{pyramid} of a right pyramid is given by:

$$V_{\text{pyramid}} = \frac{1}{3} \times (\text{Area of base}) \times (\text{Height})$$

The base $PQRS$ is a rectangle with dimensions 8 cm by 6 cm:

$$\text{Area of base} = 8 \times 6 = 48 \text{ cm}^2$$

Since we found $MV = 12$ cm, we have:

$$V_{\text{pyramid}} = \frac{1}{3} \times 48 \times 12 = \frac{576}{3} = 192 \text{ cm}^3$$

Hence, the volume of the pyramid is

$$\boxed{192 \text{ cm}^3}$$

(b)

(i) From the diagram, Q is on the 70°W meridian and T is on the 42°E meridian. The total separation in longitude is:

$$70^\circ + 42^\circ = 112^\circ$$

Hence, the difference in longitudes between Q and T is 112°

(ii) (a)

1: Identify the difference in longitude

From the diagram, the longitudes of R and P differ by 112°

2: Compute the distance per degree of longitude at 85°N

$$\text{Distance per } 1^\circ \text{ of longitude at } 85^\circ\text{N} = 60 \times \cos(85^\circ) \text{ nm}$$

Numerically, $\cos(85^\circ) \approx 0.08716$, so

$$60 \times 0.08716 \approx 5.23 \text{ nm per degree}$$

3: Multiply by the total longitude difference Since R and P differ by 112° of longitude, the arc length along 85°N is:

$$112^\circ \times 5.23 \frac{\text{nm}}{\text{degree}} \approx 586 \text{ nm}$$

Hence, the distance R to P is approximately

$$\boxed{586 \text{ nautical miles}}$$

(ii) (b)

1: Determine the difference in latitude

$$P \text{ is at } 85^\circ\text{N}, \quad T \text{ is at } 20^\circ\text{N}$$

Hence, the difference in latitude is

$$85^\circ - 20^\circ = 65^\circ$$

2: Convert degrees of latitude to nautical miles A standard approximation is that 1° of latitude corresponds to 60 nautical miles. So for 65° :

$$65^\circ \times 60 \frac{\text{nm}}{\text{degree}} = 3900 \text{ nm}$$

Q8.

(a)

1. Find the midpoint (x) of each class interval:

For each class interval, the midpoint is the average of the lower and upper class limits

Time (minutes)	Midpoint (x)
$0 < x \leq 2$	$(0 + 2) / 2 = 1$
$2 < x \leq 4$	$(2 + 4) / 2 = 3$
$4 < x \leq 6$	$(4 + 6) / 2 = 5$
$6 < x \leq 8$	$(6 + 8) / 2 = 7$
$8 < x \leq 10$	$(8 + 10) / 2 = 9$
$10 < x \leq 12$	$(10 + 12) / 2 = 11$

2. Create a table to calculate fx and fx^2 for each class interval:

Here, 'f' is the frequency (Number of learners). We need to multiply the midpoint (x) by its corresponding frequency (f) to get 'fx', and multiply 'fx' by 'x' again (or square 'x' and multiply by 'f') to get 'fx²'.

Time (minutes)	Midpoint (x)	Frequency (f)	fx	fx ²
$0 < x \leq 2$	1	5	$1 \times 5 = 5$	$1^2 \times 5 = 5$
$2 < x \leq 4$	3	15	$3 \times 15 = 45$	$3^2 \times 15 = 135$
$4 < x \leq 6$	5	25	$5 \times 25 = 125$	$5^2 \times 25 = 625$
$6 < x \leq 8$	7	20	$7 \times 20 = 140$	$7^2 \times 20 = 980$
$8 < x \leq 10$	9	10	$9 \times 10 = 90$	$9^2 \times 10 = 810$
$10 < x \leq 12$	11	5	$11 \times 5 = 55$	$11^2 \times 5 = 605$
Totals		N = 80	$\Sigma fx = 460$	$\Sigma fx^2 = 3160$

3. Calculate the mean ($\bar{x}$):

The mean for grouped data is given by the formula:

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$\bar{x} = \frac{460}{80}$$

$$\bar{x} = 5.75$$

4. Calculate the variance (σ^2):

The variance for grouped data can be calculated using the formula:

$$\sigma^2 = \left(\frac{\Sigma fx^2}{N} \right) - (\bar{x})^2$$

Alternatively, we can use:

$$\sigma^2 = \left(\frac{\Sigma fx^2}{N} \right) - \left(\frac{\Sigma fx}{N} \right)^2$$

Using the values from our table:

$$\sigma^2 = \left(\frac{3160}{80} \right) - (5.75)^2$$

$$\sigma^2 = 39.5 - (5.75)^2$$

$$\sigma^2 = 39.5 - 33.0625$$

$$\sigma^2 = 6.4375$$

5. Calculate the standard deviation (σ):

The standard deviation is the square root of the variance:

$$\sigma = \sqrt{\sigma^2}$$

$$\sigma = \sqrt{6.4375}$$

$$\sigma \approx 2.53722$$

Rounding to two decimal places (or a reasonable level of precision):

$$\sigma \approx 2.54$$

Standard Deviation = 2.54 minutes

(b) (i) Completed Cumulative Frequency Table:

The completed cumulative frequency table is:

Time (minutes)	≤ 0	≤ 2	≤ 4	≤ 6	≤ 8	≤ 10	≤ 12
Number of learners	0	5	20	45	65	75	80

Alternatively, we can use:

$$\sigma^2 = \left(\frac{\Sigma fx^2}{N} \right) - \left(\frac{\Sigma fx}{N} \right)^2$$

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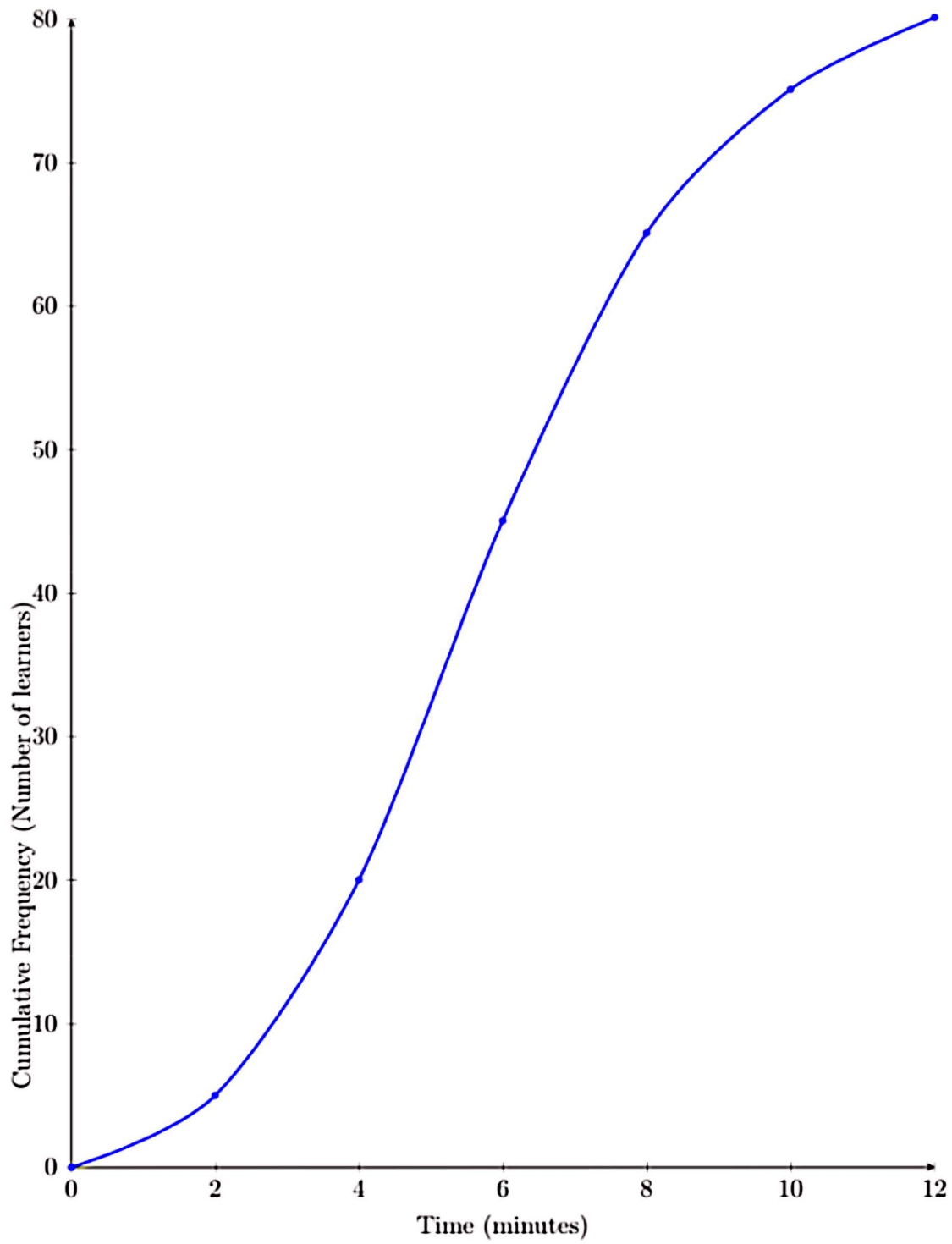
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Time (minutes)	≤ 0	≤ 2	≤ 4	≤ 6	≤ 8	≤ 10	≤ 12
Number of learners	0	5	20	45	65	75	80

(ii) Cumulative Frequency Curve (Ogive):



(iii) Estimating the Semi-Interquartile Range (SIQR):

To estimate the SIQR from the graph:

1. **Find the position of quartiles:** $Q1 \text{ position} = \frac{1}{4} \times 80 = 20$; $Q3 \text{ position} = \frac{3}{4} \times 80 = 60$
2. **Estimate Q1 from the graph:** Locate 20 on the y-axis, draw a horizontal line to the curve, and then down to the x-axis. From the cumulative frequency table, $Q1 \approx 4$ minutes
3. **Estimate Q3 from the graph:** Locate 60 on the y-axis, draw a horizontal line to the curve, and then down to the x-axis. Using interpolation, $Q3 \approx 7.5$ minutes
4. **Calculate SIQR:**

$$SIQR = \frac{Q3 - Q1}{2} = \frac{7.5 - 4}{2} = \frac{3.5}{2} = 1.75$$

The estimated semi-interquartile range is 1.75 minutes

Q9

(a) Formulate the System of Inequalities

1: Cost Constraint

Each unit of A costs \$60, and each unit of B costs \$30; the total must be at most \$3000:

$$60x + 30y \leq 3000$$

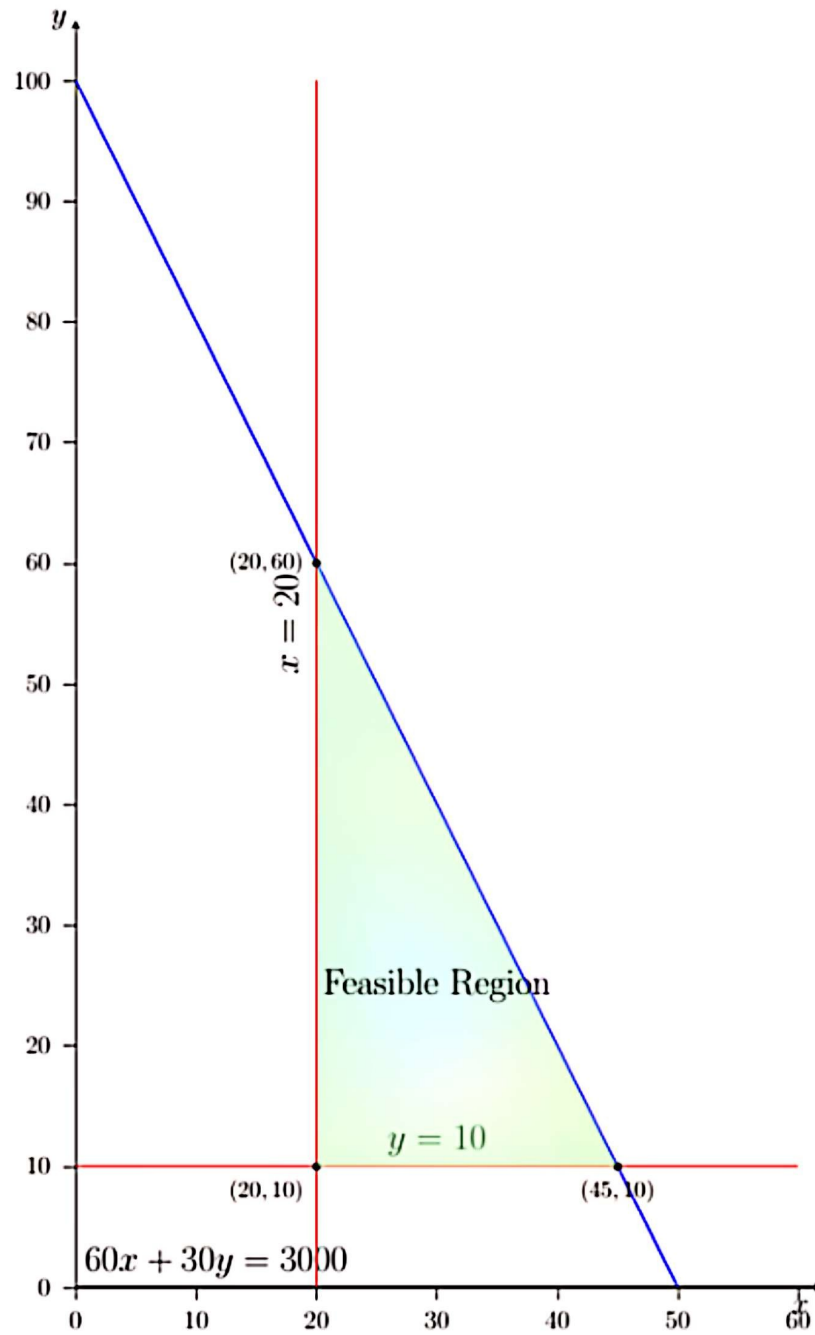
2: Minimum Requirements

$$x \geq 20, \quad y \geq 10$$

Hence, the system of inequalities is:

$$\begin{cases} 60x + 30y \leq 3000 \\ x \geq 20 \\ y \geq 10 \end{cases}$$

(b) Graphical Representation of the Feasible Region



(c) Identify the Corner Points

From the figure (or by algebraic intersection):

$$\begin{aligned}(20, 10), \\ (20, 60), \\ (45, 10)\end{aligned}$$

These are the **extreme points** (vertices) of the triangular feasible region

(d) Find the Maximum Profit

$$\text{Profit} = 80x + 50y$$

We evaluate at each of the corner points:

$$\text{Profit at } (20, 10) = 80(20) + 50(10) = 1600 + 500 = 2100,$$

$$\text{Profit at } (20, 60) = 80(20) + 50(60) = 1600 + 3000 = 4600,$$

$$\text{Profit at } (45, 10) = 80(45) + 50(10) = 3600 + 500 = 4100$$

The maximum profit is $\boxed{4600}$, attained at

$$\boxed{(x, y) = (20, 60)}$$

Q10

(a)

We are given that a clockwise rotation maps triangle JKL onto triangle ABC with:

$$J(1, 2), \quad K(2, 2), \quad L(1, 4)$$

$$A(2, -1), \quad B(2, -2), \quad C(4, -1)$$

A clockwise rotation by 90° about a point (h, k) sends a point (x, y) to

$$(h + (y - k), k - (x - h))$$

Using the mapping $J(1, 2) \rightarrow A(2, -1)$, we set:

$$\begin{cases} h + (2 - k) = 2, \\ k - (1 - h) = -1 \end{cases}$$

The first equation gives:

$$h + 2 - k = 2 \implies h - k = 0 \implies h = k$$

Substitute $h = k$ into the second equation:

$$k - (1 - k) = -1 \implies k - 1 + k = -1 \implies 2k = 0 \implies k = 0$$

Thus, $h = 0$

Verification:

$$K(2, 2) \rightarrow (0 + (2 - 0), 0 - (2 - 0)) = (2, -2) \quad (\text{matches } B),$$

$$L(1, 4) \rightarrow (0 + (4 - 0), 0 - (1 - 0)) = (4, -1) \quad (\text{matches } C)$$

Centre $(0, 0)$ and angle 90° clockwise
--

(b)

The transformation is given by the matrix

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

This transformation sends any point (x, y) to $(-2x, -2y)$. Note that multiplying by -2 reflects the point through the origin and enlarges distances by a factor of 2

(i) **Scale Factor:** The scale factor is the absolute value of -2 , i.e.,

$$\boxed{2}$$

(ii) **Coordinates of X, Y, Z :** Triangle JKL has vertices:

$$J(1, 2), \quad K(2, 2), \quad L(1, 4)$$

Their images are:

$$X = (-2 \cdot 1, -2 \cdot 2) = (-2, -4),$$

$$Y = (-2 \cdot 2, -2 \cdot 2) = (-4, -4),$$

$$Z = (-2 \cdot 1, -2 \cdot 4) = (-2, -8)$$

$$\boxed{X(-2, -4), \quad Y(-4, -4), \quad Z(-2, -8)}$$

(c)

Triangle JKL is mapped onto triangle PQR where:

$$P(4, 2), \quad Q(8, 2), \quad R(4, 4)$$

and the original vertices are:

$$J(1, 2), \quad K(2, 2), \quad L(1, 4)$$

Assume the transformation is represented by the matrix

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

$$\text{such that } M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

Using the mappings:

1. $J(1, 2) \rightarrow P(4, 2)$:

$$\begin{cases} a + 2b = 4, \\ c + 2d = 2 \end{cases}$$

2. $K(2, 2) \rightarrow Q(8, 2)$:

$$\begin{cases} 2a + 2b = 8, \\ 2c + 2d = 2 \end{cases}$$

Dividing by 2 gives:

$$\begin{cases} a + b = 4, \\ c + d = 1 \end{cases}$$

3. $L(1, 4) \rightarrow R(4, 4)$:

$$\begin{cases} a + 4b = 4, \\ c + 4d = 4 \end{cases}$$

Solving for a and b :

$$a + b = 4 \implies a = 4 - b$$

Substitute into $a + 2b = 4$:

$$(4 - b) + 2b = 4 \implies 4 + b = 4 \implies b = 0$$

Thus, $a = 4$

Solving for c and d :

$$c + d = 1 \implies c = 1 - d$$

Substitute into $c + 2d = 2$:

$$(1 - d) + 2d = 2 \implies 1 + d = 2 \implies d = 1$$

Thus, $c = 0$

The transformation matrix is

$$\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

which represents a stretch in the x -direction by a factor of 4

(d)

Triangle JKL is mapped onto triangle DEF where:

$$D(-3, 2), \quad E(-2, 2), \quad F(-7, 4)$$

and the original vertices are:

$$J(1, 2), \quad K(2, 2), \quad L(1, 4)$$

Observation: Notice that the y -coordinates remain unchanged:

$$J(1, 2) \rightarrow D(-3, 2), \quad K(2, 2) \rightarrow E(-2, 2), \quad L(1, 4) \rightarrow F(-7, 4)$$

However, the x -coordinates shift by an amount that depends on the y -value:

$$J : 1 \rightarrow -3 \quad (\text{shift } -4),$$

$$K : 2 \rightarrow -2 \quad (\text{shift } -4),$$

$$L : 1 \rightarrow -7 \quad (\text{shift } -6 \text{ compared to } 1, \text{ but note: } 1 - 2(4) = -7)$$

This suggests a shear transformation.

A shear parallel to the x -axis is given by:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + ky \\ y \end{pmatrix}$$

Using $J(1, 2) \rightarrow D(-3, 2)$, we require:

$$1 + k(2) = -3 \implies 1 + 2k = -3 \implies 2k = -4 \implies k = -2$$

Verification:

$$K(2, 2) \rightarrow (2 + (-2)(2), 2) = (2 - 4, 2) = (-2, 2) \quad (\text{matches } E),$$

$$L(1, 4) \rightarrow (1 + (-2)(4), 4) = (1 - 8, 4) = (-7, 4) \quad (\text{matches } F)$$

The single transformation is a shear parallel to the x -axis with shear factor -2 , represented by the matrix:

$$\boxed{\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}}$$

This shear leaves the y -coordinate unchanged and shifts the x -coordinate by $-2y$

Q11

(a)(i) Finding k

The curve is given by

$$y = x^3 - 6x + 2$$

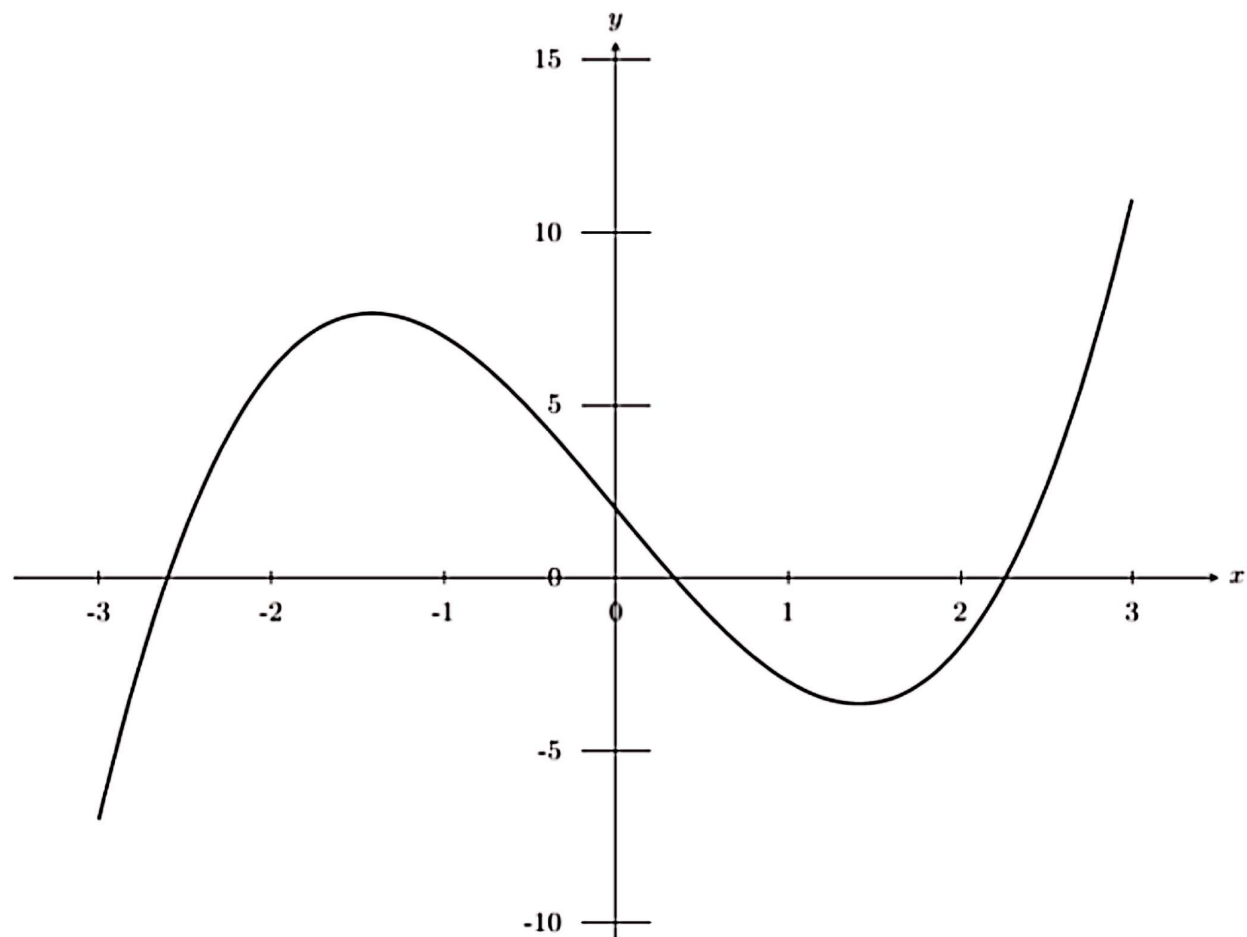
When $x = 3$,

$$y = 3^3 - 6 \cdot 3 + 2 = 27 - 18 + 2 = 11$$

Thus,

$$\boxed{k = 11}$$

(a)(ii)



(a)(iii)(a) Using the Graph to Solve $x^3 - 6x + 2 = 0$

The solutions of the equation

$$x^3 - 6x + 2 = 0$$

are the x -coordinates where the curve crosses the x -axis (i.e. where $y = 0$).
Reading from the graph, the approximate solutions are:

$$x \approx -2.6, \quad 0.3, \quad 2.3$$

(a)(iii)(b) Using the Graph to Solve $x^3 - 6x = 2$

Rewrite the equation as follows:

$$x^3 - 6x = 2 \quad \Longleftrightarrow \quad x^3 - 6x - 2 = 0$$

An equivalent form is

$$x^3 - 6x + 2 = 4$$

Thus, on the graph of $y = x^3 - 6x + 2$ we now draw the horizontal line $y = 4$ and look for its intersections with the curve

From the graph, the approximate solutions are:

$$x \approx -2.3, \quad -0.3, \quad 2.6$$

(b)

1. Differentiate the curve $y = x^3 + 3x^2 + 2$:

$$\frac{dy}{dx} = 3x^2 + 6x$$

2. Evaluate the derivative at the point $(-1, 4)$ to find the slope of the tangent:

$$m_{\text{tangent}} = 3(-1)^2 + 6(-1) = 3 - 6 = -3$$

3. Find the slope of the normal, which is the negative reciprocal of the slope of the tangent:

$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = -\frac{1}{-3} = \frac{1}{3}$$

4. Use the point-slope form for the line passing through $(-1, 4)$ with slope $\frac{1}{3}$:

$$y - 4 = \frac{1}{3}(x - (-1)) = \frac{1}{3}(x + 1)$$

5. Simplify to get the slope-intercept form:

$$y - 4 = \frac{1}{3}x + \frac{1}{3},$$

$$y = \frac{1}{3}x + \frac{1}{3} + 4 = \frac{1}{3}x + \frac{1}{3} + \frac{12}{3} = \frac{1}{3}x + \frac{13}{3}$$

Hence, the equation of the normal is:

$$\boxed{y = \frac{1}{3}x + \frac{13}{3}}$$

Q12

(a)

Given:

$$PQ = 16.1 \text{ m}, \quad \angle QPR = 42^\circ, \quad \angle PQR = 53^\circ$$

Since the angles of a triangle sum to 180° :

$$\angle PRQ = 180^\circ - (42^\circ + 53^\circ) = 85^\circ$$

(i) Find QR :

Use the Sine Rule:

$$\frac{PQ}{\sin(\angle PRQ)} = \frac{QR}{\sin(\angle QPR)} \implies QR = PQ \frac{\sin(42^\circ)}{\sin(85^\circ)}$$

$$QR = 16.1 \times \frac{\sin(42^\circ)}{\sin(85^\circ)} \approx 16.1 \times \frac{0.6691}{0.9962} \approx 10.8 \text{ m}$$

(ii) Find the area of $\triangle PQR$:

$$\text{Area} = \frac{1}{2} (PQ) (QR) \sin(\angle PQR)$$

$$\text{Area} = \frac{1}{2} \times 16.1 \times 10.8 \times \sin(53^\circ) \approx \frac{1}{2} \times 16.1 \times 10.8 \times 0.7986 \approx 69.5 \text{ m}^2$$

(iii) Shortest distance from R to PQ :

1. Recall that the shortest distance is the altitude: If h is the altitude from R onto side PQ , then

$$\text{Area} = \frac{1}{2} \times (PQ) \times h$$

2. Solve for h :

$$h = \frac{2 \times \text{Area}}{PQ} = \frac{2 \times 69.5}{16.1} \approx 8.6 \text{ m}$$

$QR \approx 10.8 \text{ m}, \quad \text{Area} \approx 69.5 \text{ m}^2, \quad \text{Altitude} \approx 8.6 \text{ m}$
--

(b)

1. Isolate $\sin \theta$:

$$6 \sin \theta = -3 \implies \sin \theta = -\frac{1}{2}$$

2. Recall where $\sin \theta = -\frac{1}{2}$: The reference angle for $\sin \theta = \frac{1}{2}$ is 30° , but we need negative sine (i.e., below the x-axis) which occurs in the third and fourth quadrants.

3. Hence solutions in $[180^\circ, 360^\circ]$:

$$\theta = 180^\circ + 30^\circ = 210^\circ, \quad \text{or} \quad \theta = 360^\circ - 30^\circ = 330^\circ$$

$\theta = 210^\circ \quad \text{or} \quad 330^\circ$
--

(c)

1. Factor the numerator:

$$x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$$

2. Simplify by canceling $(x + 3)$ (assuming $x \neq -3$):

$$\frac{x^3 - 9x}{x + 3} = \frac{x(x - 3)(x + 3)}{x + 3} = x(x - 3)$$

$$\boxed{\frac{x^3 - 9x}{x + 3} = x(x - 3), \quad \text{for } x \neq -3}$$