SUGGESTED ANSWERS

Q1.

(a) $\frac{m-4}{(3m-2)(2m-3)}$

1. Denominators:

$$\frac{2}{3m-2}-\frac{1}{2m-3}$$

2. Common denominator (LCD):

$$(3m-2)(2m-3)$$

3. Rewrite each fraction over the LCD:

$$\frac{2}{3m-2}=\frac{2(2m-3)}{(3m-2)(2m-3)}$$
$$\frac{1}{2m-3}=\frac{1(3m-2)}{(3m-2)(2m-3)}$$

4. Subtract numerators:

$$2(2m-3)-1(3m-2)=4m-6-(3m-2)=4m-6-3m-2=m-4$$

5. Result in lowest terms:

$$\frac{m-4}{(3m-2)(2m-3)}$$

So

$$\frac{2}{3m-2}-\frac{1}{2m-3}=\frac{m-4}{(3m-2)(2m-3)}$$

(b) $x \approx 1.33,\ x \approx -1.76$

We solve the quadratic

$$x^2 = 3x^2 + 1.3x - 7 = 0$$

using the formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1. Identify coefficients

$$a = 3, b = 1.3, c = -7$$

2. Compute the discriminant

$$\Delta = b^2 - 4ac = (1.3)^2 - 4 \cdot 3 \cdot -7 = 1.69 + 84 = 85.69$$

3. Take the square root

$$\sqrt{\Delta} = \sqrt{85.69} \approx 9.2569.$$

4. Apply the quadratic formula

$$x = \frac{-1.3 \pm 9.2569}{2 \cdot 3} = \frac{-1.3 \pm 9.2569}{6}$$

5. Compute each root

$$x_1 = \frac{-1.3 + 9.2569}{6} = \frac{7.9569}{6} \approx 1.3262 \Rightarrow 1.33$$

$$x_2 = \frac{-1.3 - 9.2569}{6} = \frac{-}{10.5569} \approx -1.7595 \Rightarrow -1.76$$

$$\boxed{x \approx 1.33, \qquad x \approx -1.76}$$

Q2.

(a)(i) $x = -2$

1. Compute the determinant:

$$\det(K) = (8)(x) - (9x)(1) = 8x - 9x = -x$$

2. Set -x = 2, so $x = -2$

(a)(ii) $K^{-1} = \begin{pmatrix} -1 & 9 \\ \frac{1}{x} & -\frac{8}{x} \end{pmatrix}, x \neq 0$

1. Determinant in general:

$$\det(K) = -x \Rightarrow K \text{ is invertible iff} x \neq 0$$

2. Adjugate matrix: swap the diagonal entries and negate the off-diagonals:

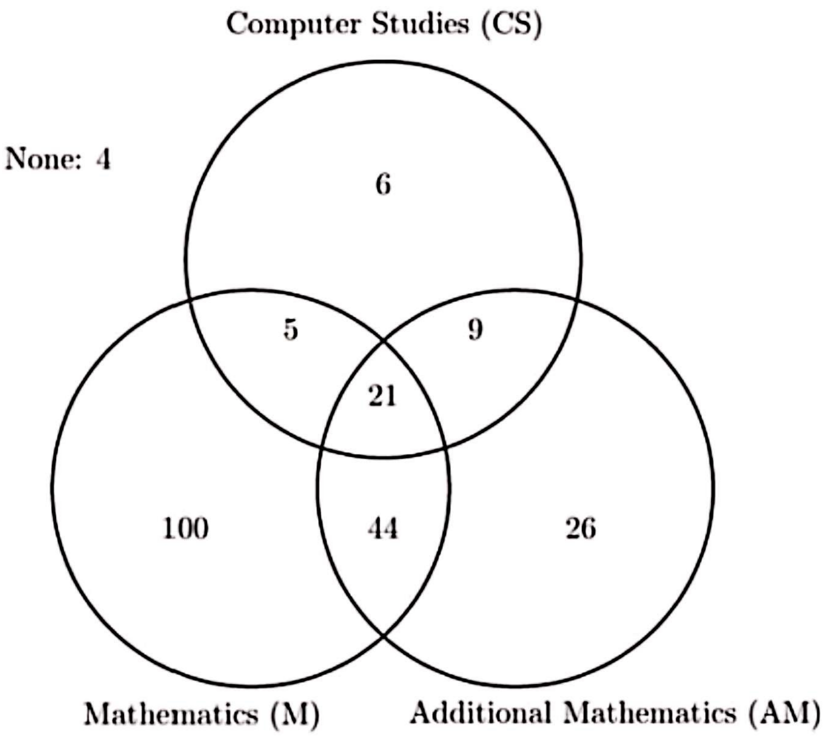
$$\text{adj}(K) = \begin{pmatrix} x & -9x \\ -1 & 8 \end{pmatrix}$$

3. Inverse formula:

$$K^{-1} = \frac{1}{\det(K)} \operatorname{adj}(K) = \frac{1}{-x} \begin{pmatrix} x & -9x \\ -1 & 8 \end{pmatrix} = \begin{pmatrix} -1 & 9 \\ \frac{1}{x} & -\frac{8}{x} \end{pmatrix}$$

valid for $x \neq 0$

(b)(i)



Total teachers: 215

(b)(ii)(a) 4

$$215 - 211 = \boxed{4}$$

(b)(ii)(b) 41

$$(\text{only A}) + (\text{only C}) + (A \cap C\text{only}) = 26 + 6 + 9 = \boxed{41}$$

(b)(ii)(c) 58

$$(M \cap A\text{only}) + (M \cap C\text{only}) + (A \cap C\text{only}) = 44 + 5 + 9 = \boxed{58}$$

Q3.

(a) $y = 3x^4 - 2x^2 + 5x - 2$

1. We have

$$\frac{dy}{dx} = 12x^3 - 4x + 5$$

2. Integrate both sides w.r.t. x :

$$y = \int (12x^3 - 4x + 5)dx = \int 12x^3 dx - \int 4x dx + \int 5dx$$

3. Compute each integral:

- $\int 12x^3 dx = 12 \cdot \frac{x^4}{4} = 3x^4$
- $\int 4x dx = 4 \cdot \frac{x^2}{2} = 2x^2$
- $\int 5dx = 5x$

So

$$y = 3x^4 - 2x^2 + 5x + C$$

4. Use the point $(0, -2)$ to find C:

$$y(0) = 3\cdot 0^4 - 2\cdot 0^2 + 5\cdot 0 + C = C = -2 \Rightarrow C = -2$$

5. Final equation of the curve:

$$\boxed{y = 3x^4 - 2x^2 + 5x - 2}$$

(b) (i) 22.5

$$t_1 = 180\frac{1^1}{2} = 180 \cdot \frac{1}{2} = 90,$$

$$t_2 = 180\frac{1^2}{2} = 180 \cdot \frac{1}{4} = 45,$$

$$t_3 = 180\frac{1^3}{2} = 180 \cdot \frac{1}{8} = 22.5$$

(b) (ii) $\frac{1}{2}$

$$r = \frac{t_{n+1}}{t_n} = \frac{180\frac{1}{2}^{n+1}}{180\frac{1}{2}^{n}} = \frac{1}{2}$$

(b) (iii) 180

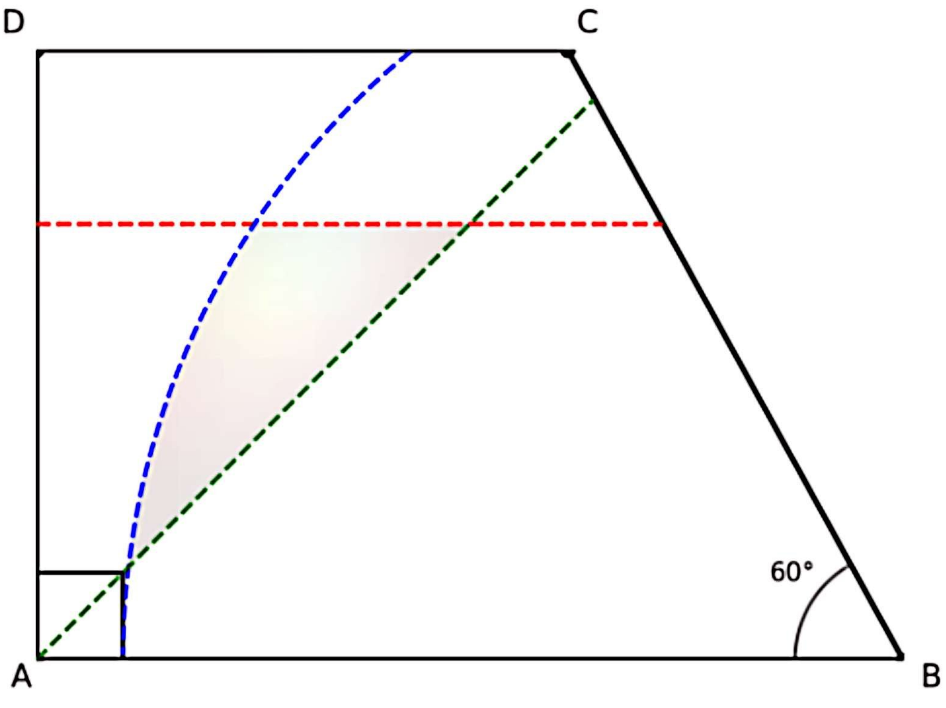
A geometric series converges for $|r| < 1$.Here $r = \frac{1}{2}$, so

$$S_\infty = \frac{a}{1-r},$$

where $a = t_1 = 90$. Thus

$$S_\infty = \frac{90}{1-\frac{1}{2}} = \frac{90}{1-\frac{1}{2}} = 180$$

Q4.



Q5.

(a) $\frac{q}{5p}$

1.Rewrite as multiplication by the reciprocal

$$\frac{10p^3q^4}{8m^4n^2} \div \frac{25p^4q^3}{4m^4n^2} = \frac{10p^3q^4}{8m^4n^2} \times \frac{4m^4n^2}{25p^4q^3}$$

2. Separate coefficients and like bases

$$= \frac{10 \times 4}{8 \times 25} \times \frac{p^3}{p^4} \times \frac{q^4}{q^3} \times \frac{m^4}{m^4} \times \frac{n^2}{n^2}$$

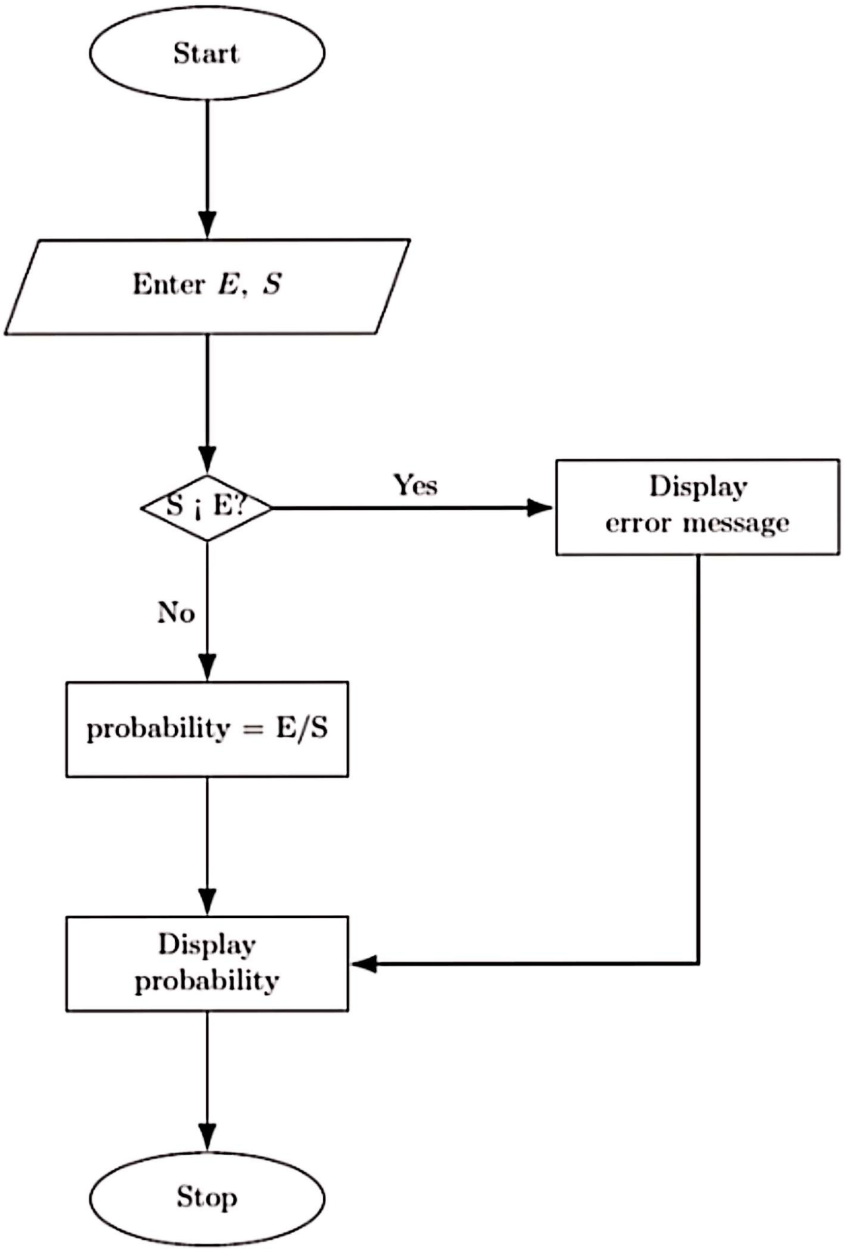
3. Simplify each part

- Coefficients: $\frac{10 \cdot 4}{8 \cdot 25} = \frac{40}{200} = \frac{1}{5}$
- Powers of p: $p^{3-4} = p^{-1} = \frac{1}{p}$
- Powers of q: $q^{4-3} = q^1 = q$

Step 4. Combine into lowest terms

$$\frac{1}{5} \times \frac{q}{p} = \boxed{\frac{q}{5p}}$$

(b)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Enter E, S
S ¡ E?
Yes
Display error message
No
probability = E/S
Display probability
Stop

Q6.

(a)(i) $\frac{5}{18}$

- Total pens: 9 (5 blue, 4 black).
- Probability first is blue: $\frac{5}{9}$.
- Given first blue, remaining blue pens = 4, total pens = 8, so second is blue: $\frac{4}{8}$

$$P(\text{both blue}) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72} = \frac{5}{18}$$

(a)(ii) $\frac{4}{9}$

This is the sum of "both blue" and "both black."

- We already have $P(\text{both blue}) = \frac{5}{18}$
- Now "both black":

$$P(\text{first black}) = \frac{4}{9}, P(\text{ second black} \mid \text{first black }) = \frac{3}{8}$$

so

$$P(\text{both black}) = \frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$$

Hence

$$P(\text{same colour}) = P(\text{both blue}) + P(\text{both black}) = \frac{5}{18} + \frac{1}{6} = \frac{5}{18} + \frac{3}{18} = \frac{8}{18} = \frac{4}{9}$$

(b)(i) $3c - 2a$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 3c - 2a$$

(b)(ii) $a + \frac{3}{2}c$

$$\overrightarrow{OD} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC} = 2a + \frac{1}{2}(3c - 2a) = a + \frac{3}{2}c$$

(b)(iii) $2a + 9c$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 2a + 3\overrightarrow{OC} = 2a + 3(3c) = 2a + 9c$$

(b)(iv) $-a - \frac{15}{2}c$

$$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB} = (a + \frac{3}{2}c) - (2a + 9c) = -a - \frac{15}{2}c$$

Q7.

(a)(i) $100°$

$$50° + 50° = 100°$$

(a)(ii) (a) 4,288km $(along\ 50°N)$

Use $R = 6370\text{km}$ and $\pi = 3.142$

Along latitude 50° N, from A (20° W) to B (40° E)

- Change in longitude $\Delta\lambda = 40° - (-20°) = 60°$
- Radius of the parallel at latitude $\varphi = 50°$ is $R_{\parallel} = R\cos\varphi = 6370\cos 50°$
- Convert $\Delta\lambda$ to radians: $\Delta\lambda_{\text{rad}} = 60° \times \frac{\pi}{180} = 60 \times \frac{3.142}{180} = 1.0473\text{rad}$
- $d_{AB} = R_{\parallel}\Delta\lambda_{\text{rad}} = 6370\cos 50° \times 1.0473 \approx 6370 \times 0.6429 \times 1.0473 \approx 4288\text{km}$

(a)(ii) (b) 11,120km $(along\ 40°E)$

Along longitude 40° E, from B (50° N) to C (50° S)

- Change in latitude $\Delta\varphi = 50° - (-50°) = 100°$
- Convert $\Delta\varphi$ to radians: $\Delta\varphi_{\text{rad}} = 100° \times \frac{\pi}{180} = 100 \times \frac{3.142}{180} = 1.7456 \text{ rad}$
- $d_{BC} = R\Delta\varphi_{\text{rad}} = 6370 \times 1.7456 \approx 11120\text{km}$

(b)(i) 24cm

By similar cross-sections, if the full pyramid height is H, then at the cut

$$\frac{\text{top length}}{\text{base length}} = \frac{10}{15} = \frac{2}{3} = \frac{H-8}{H} \Rightarrow 3(H-8) = 2H \Rightarrow H = 24\text{cm}$$

(b)(ii) 506.7cm³

Volume of frustum = volume of big pyramid - volume of small "tip" pyramid above the cut.

- Big pyramid:

$$V_{\text{big}} = \frac{1}{3}(15 \cdot 6)H = \frac{1}{3} \cdot 90 \cdot 24 = 720\text{cm}^3$$

- Small top pyramid (base 10 × 4, height = H - 8 = 16):

$$V_{\text{small}} = \frac{1}{3}(10 \cdot 4) \cdot 16 = \frac{1}{3} \cdot 40 \cdot 16 = \frac{640}{3}\text{cm}^3$$

- Frustum:

$$V = 720 - \frac{640}{3} = \frac{2160}{3} - \frac{640}{3} = \frac{1520}{3} \approx 506.7\text{cm}^3$$

Or directly with the frustum formula

$$V = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 \cdot A_2}\right)$$

with $h = 8, A_1 = 15 \cdot 6 = 90, A_2 = 10 \cdot 4 = 40$, gives the same $\frac{1520}{3}$

Q8.

(a)

A single half-turn (rotation of 180°) about the origin

Under this rotation $(x, y) \mapsto (-x, -y)$, so

$$A(1,2) \to A_1(-1,-2), B(3,2) \to B_1(-3,-2), C(3,4) \to C_1(-3,-4)$$

(b)

Under the enlargement with centre (1, 0) and scale factor 2, a point (x, y) goes to

$$(1 + 2(x - 1), 0 + 2(y - 0)) = (2x - 1, 2y)$$

Thus

$$A_2 = (2 \cdot 1 - 1, 2 \cdot 2) = (1,4), B_2 = (2 \cdot 3 - 1, 2 \cdot 2) = (5,4), C_2 = (2 \cdot 3 - 1, 2 \cdot 4) = (5,8)$$

(c)

Looking at the two triangles, the side through B_1C_1 remains fixed in position (it is carried onto B_3C_3 which is the same vertical line), so

1.Invariant line: the lineB_1C_1(the vertical line through those two points).

2.Scale factor:

$$\frac{|B_3C_3|}{|B_1C_1|} = \frac{8-4}{(-2)-(-4)} = \frac{4}{2} = 2$$

(d)

The diagram shows a horizontal shear mapping

$$(x, y) \mapsto (x + ky, y)$$

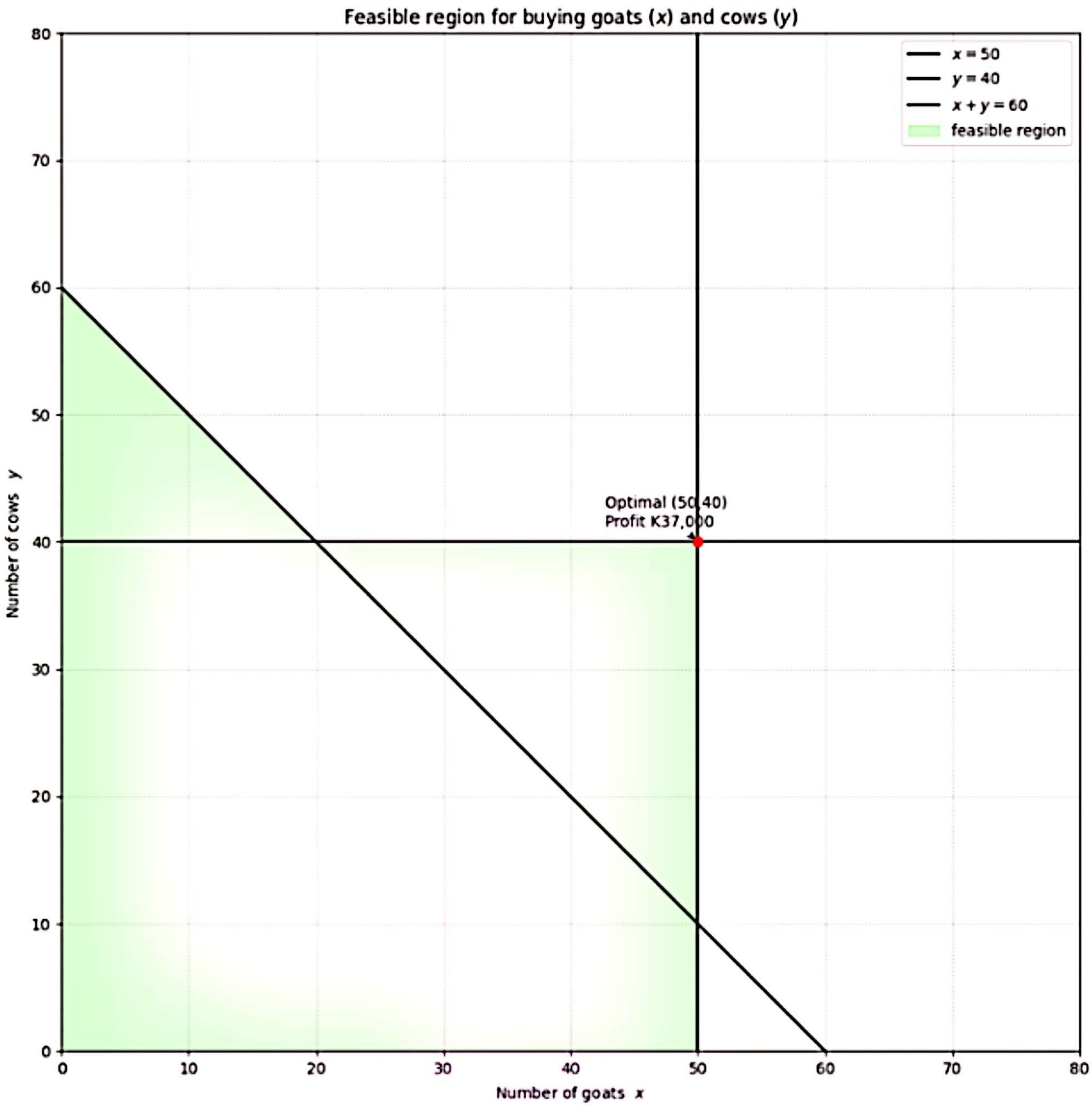
and one reads off from, say,

$$A(1,2) \mapsto A_4(5,2) \Rightarrow 1 + 2k = 5 \Rightarrow k = 2$$

Hence the shear matrix is

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

Q9



(a) $x \leq 50, y \leq 40, x + y \geq 60, x \geq 0, y \geq 0$

(c) Buy 50 goats and 40 cows

(d) Maximum profit = K 37,000

Q10

(a)(i)

To find q, evaluate

$$y = x^3 - 9x + 5$$

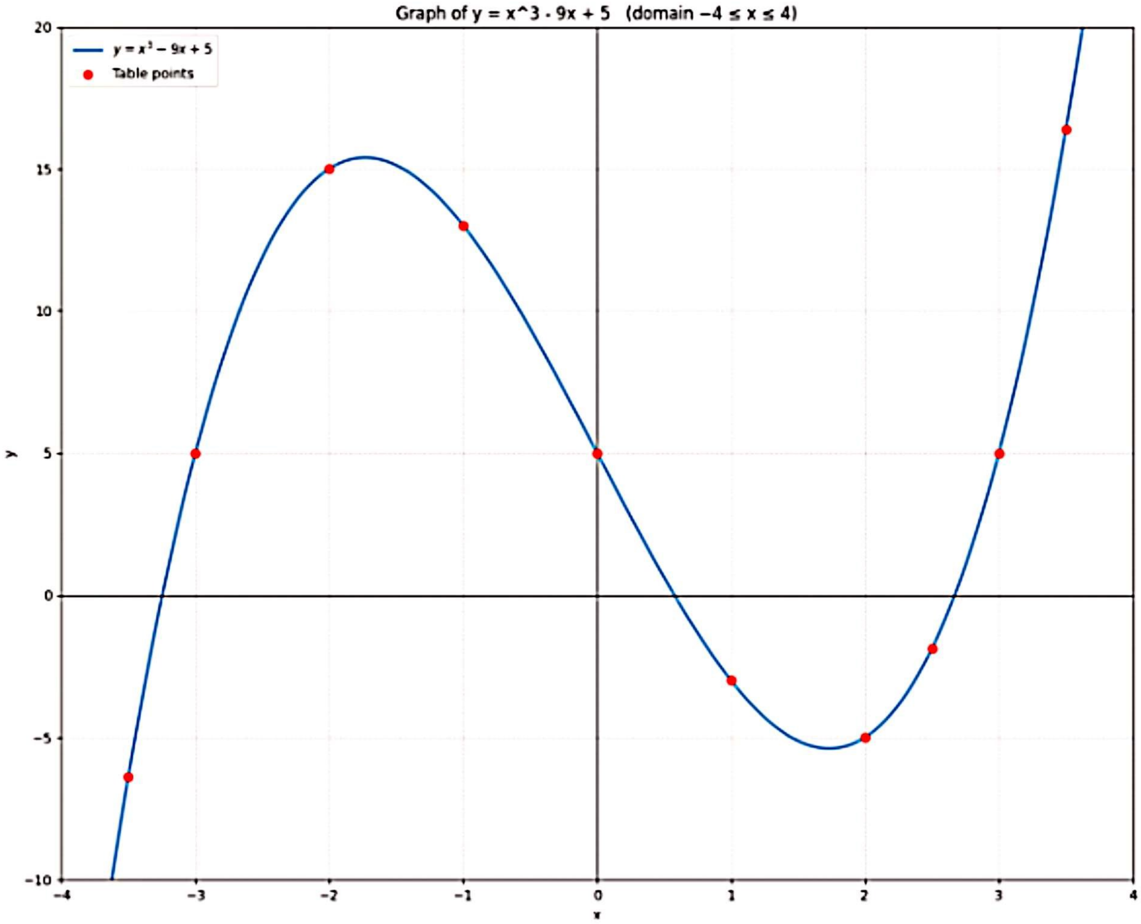
at $x = -2$:

1. Compute the cube:$(2)^3 = -8$
2. Compute the linear term: $-9 \times (-2) = +18$
3. Sum all parts:$y = -8 + 18 + 5 = 10 + 5 = 15$

Hence

$$\boxed{q = 15}$$

(a)(ii)



(iii)(a)

The curve crosses the $x_{\text{-axis}}$ at approximately

$$\boxed{x \approx -3.25, \qquad 0.57, \qquad 2.68}$$

(iii)(b)

$\Rightarrow x^3 - 6x - 1 = 0$. The three real roots are

$$\boxed{x \approx -2.36, \qquad -0.17, \qquad 2.53}$$

Q11

(a) $\angle QPR \approx 39.9°$

In ΔPQR we have

$$PQ = 41.6, PR = 70, \angle PQR = 105°$$

First use the sine rule to find ∠PRQ (i.e. ∠R):

$$\frac{\sin R}{\text{PQ}} = \frac{\sin Q}{\text{PR}} \Rightarrow \sin R = \frac{\text{PQ}}{\text{PR}} \sin Q = \frac{41.6}{70} \sin 105° \approx 0.5943 \times 0.9659 \approx 0.5744$$

Hence $R = \arcsin(0.5744) \approx 35.09°$

Finally

$$P = 180° - Q - R = 180° - 105° - 35.09° \approx 39.91°.$$

So

$$\boxed{\angle QPR \approx 39.9°}$$

(b) Area ≈ 938m²

We can use

$$\text{Area} = \frac{1}{2}(\text{PQ})(\text{QR})\sin\angle\text{PQR}.$$

But first find QR by the sine rule:

$$\frac{\text{QR}}{\sin P} = \frac{\text{PR}}{\sin Q} \Rightarrow \text{QR} = \frac{\text{PR}}{\sin Q}\sin P = \frac{70}{\sin 105°}\sin 39.91° \approx \frac{70}{0.9659} \times 0.6428 \approx 46.60\text{m}.$$

Then

$$\text{Area} = \frac{1}{2} \cdot 41.6 \cdot 46.60 \cdot \sin 105° \approx 0.5 \times 41.6 \times 46.60 \times 0.9659 \approx 938\text{m}^2$$

So

$$\boxed{\text{Area} \approx 938\text{m}^2}$$

(c) Distance ≈ 26.8m

The altitude h from Q onto base PR satisfies

$$\text{Area} = \frac{1}{2}(\text{PR})\,h \Rightarrow h = \frac{2}{\text{PR}}(\text{Area}) = \frac{2 \times 938}{70} \approx 26.8\,\text{m}.$$

Thus

$$\boxed{\text{Distance} \approx 26.8\text{ m}}$$

Q12

(a) $\sigma \approx 13.0$ **years**

Step 1 .Find class mid-points and then $\sum f, \sum fx, \sum fx^2$:

Class	*f*	*mid − point x*	*f x*	x^2	fx^2
0 – 10	2	5	10	25	50
10 – 20	6	15	90	225	1350
20 – 30	9	25	225	625	5625
30 – 40	7	35	245	1225	8575
40 – 50	4	45	180	2025	8100
50 – 60	2	55	110	3025	6050
Totals	**30**		**860**		**29750**

Step 2. Compute the mean $\overline{x}$:

$$\frac{\sum fx}{\sum f} = \frac{860}{30} \approx 28.67$$

Step 3. Use the shortcut formula for the population variance

$$\sigma^2 = \frac{\sum fx^2}{\sum f} - \overline{x}^2 = \frac{29750}{30} - (28.67)2 \quad \approx 991.67 - 821.78 = 169.89$$

Step 4. Take the square root for the standard deviation

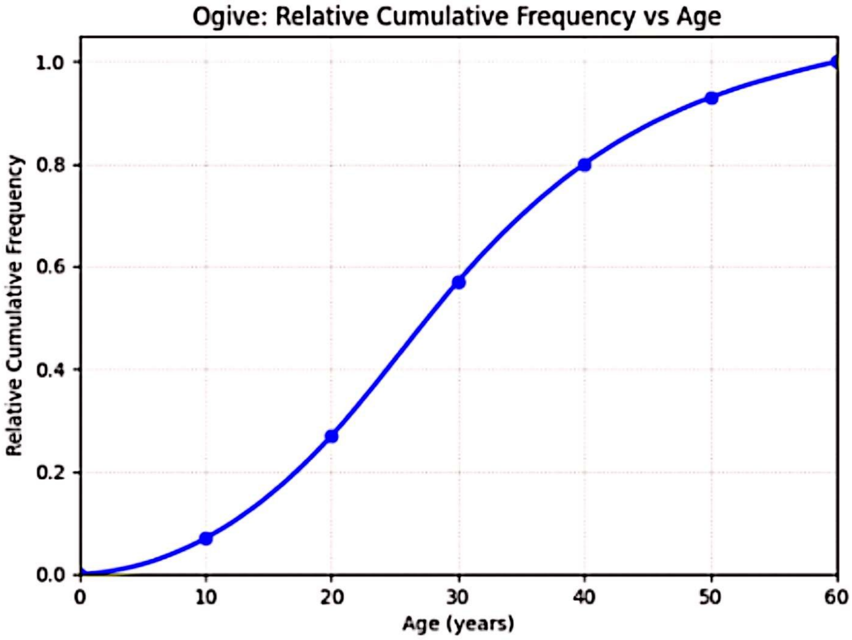
$$\sigma = \sqrt{169.89} \approx 13.03$$

$$\boxed{\sigma \approx 13.0 \text{ years}}$$

(b)(i)

Age (years)	**≤ 0**	**≤ 10**	**≤ 20**	**≤ 30**	**≤ 40**	**≤ 50**	**≤ 60**
Cumulative frequency	**0**	**2**	**8**	**17**	**24**	**28**	**30**
Relative cumulative frequency	**0.00**	**0.07**	**0.27**	**0.57**	**0.80**	**0.93**	**1.00**

(b)(ii)



(b)(iii) 36 years (to the nearest year)

To estimate the 70th percentile:

1. On the vertical axis locate $y = 0.70$.
2. Draw a horizontal line from 0.70 until it meets the ogive.
3. From that intersection drop a vertical down to the age (horizontal) axis.

Interpolation between your plotted points (at 30 years → 0.57, and 40 years → 0.80):

$$x \approx 30 + (40 - 30) \cdot \frac{0.70 - 0.57}{0.80 - 0.57} = 30 + 10 \cdot \frac{0.13}{0.23} \approx 30 + 5.65 \approx 35.7 \text{years.}$$

So the 70th percentile is approximately

$$\boxed{\text{36 years (to the nearest year)}}$$