SUGGESTED ANSWERS

Q1. $\frac{2}{3}$

$(\frac{8}{27})^{\frac{1}{3}} = \frac{8^{\frac{1}{3}}}{27^{\frac{1}{3}}}$ (use$(a/b)^n = \frac{a^n}{b^n}$)

$= \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$ (rewrite $x^{\frac{1}{3}}$ as cube root)

$= \frac{2}{3}(2^3 = 8, 3^3 = 27)$

So, $(\frac{8}{27})^{\frac{1}{3}} = \frac{2}{3}$

Q2. $3x(x-3)(x+3)$

$3x^3 - 27x = 3x(x^2 - 9)$(factor out the common$3x$)

$= 3x(x^2 - 3^2)$(note$9 = 3^2$)

$= 3x(x-3)(x+3)$(factor the difference of squares)

$\boxed{3x(x-3)(x+3)}$

Q3. $\frac{2}{a} - 4ab + 5b^2$

$2a^{-1} - 5b(a-b) + ab = \frac{2}{a} - 5b(a-b) + ab$ (rewrite$2a^{-1}$as$\frac{2}{a}$)

$= \frac{2}{a} - 5ab + 5b^2 + ab$ (expand $-5b(a-b)$)

$= \frac{2}{a} - 4ab + 5b^2$ (combine $-5ab + ab = -4ab$)

$\boxed{\frac{2}{a} - 4ab + 5b^2}$

Q4. $B = (13, -7)$

1.Midpoint formula:

$$\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = (4, -2)$$

2.For the x-coordinate:

$$\frac{-5 + x_B}{2} = 4 \Longrightarrow -5 + x_B = 8 \Longrightarrow x_B = 13$$

3.For the y-coordinate:

$$\frac{3 + y_B}{2} = -2 \Longrightarrow 3 + y_B = -4 \Longrightarrow y_B = -7$$

So $B = (13, -7)$

Q5. $|\vec{PQ}| = 13$

1.Coordinates of P and Q

$$P = (4,13), Q = (16,8)$$

2. Vector $\vec{PQ}$

$$\vec{PQ} = Q - P = (16 - 4, 8 - 13) = (12, -5)$$

3. Magnitude $|\vec{PQ}|$

$$|(12, -5)| = \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

So $|\vec{PQ}| = 13$

Q6. $\boxed{(P \cap Q) \cup (P \cap R)}$ **or** $\boxed{P \cap (Q \cup R)}$

The shaded region is everything that lies in set P and also in either Q or R.

In set notation you can write it in either of these equivalent ways:

$\boxed{(P \cap Q) \cup (P \cap R)}$ or $\boxed{P \cap (Q \cup R)}$.

Q7.

(a) $A^T = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

Given

$$A = (1 \quad 2 \quad 4),$$

the transpose A^T is a 3×1 column vector:

$$A^T = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

(b) $AB = 19$

We have$A$as a 1×3 row and

$$B = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

as a 3×1 column, so AB is a 1×1 matrix (i.e. a scalar):

$$AB = (1 \quad 2 \quad 4)\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = 1 \cdot 3 + 2 \cdot 0 + 4 \cdot 4 = 3 + 0 + 16 = 19$$

So

$$\boxed{AB = 19}$$

Q8.

(a) 19

1. Identify given terms

$$a_1 = 25, a_3 = 13.$$

2. Use the formula for the third term

$$a_3 = a_1 + 2d \Longrightarrow 25 + 2d = 13 \Longrightarrow 2d = 13 - 25 = -12 \Longrightarrow d = -6.$$

3. Second term

$$a_2 = a_1 + d = 25 + (-6) = 19$$

(b) $a_n = 31 - 6n$

1. General formula for the nth term

$$a_n = a_1 + (n-1)d = 25 + (n-1)(-6) = 25 - 6(n-1)$$

2. You can also rewrite this as

$$a_n = 31 - 6n$$

Q9

(a) $\frac{1}{3}$

1. List the letters of "EXCELLENT":

$$E, X, C, E, L, L, E, N, T$$

So there are9 letters in total.

2. Count how many of these are "E":

$$E \text{ appears at positions} 1,4,7 \Longrightarrow 3 \text{times.}$$

3. Probability of picking "E" is

$$P(E) = \frac{\text{number of E's}}{\text{total letters}} = \frac{3}{9} = \frac{1}{3}$$

$$\boxed{\frac{1}{3}}$$

(b) 2.4cm

1. Let the linear scale factor from the larger shape down to the smaller be$k$, so

$$(\frac{\text{area}_{\text{small}}}{\text{area}_{\text{large}}}) = k^2 = \frac{25}{36}$$

2. Hence

$$k = \sqrt{\frac{25}{36}} = \frac{5}{6},$$

which means

$$\frac{\text{length}_{\text{small}}}{\text{length}_{\text{large}}} = \frac{5}{6} \Rightarrow \text{length}_{\text{large}} = \frac{6}{5}\text{length}_{\text{small}}.$$

3. Since $\text{length}_{\text{small}} = 2\text{cm}$,

$$\text{length}_{\text{large}} = \frac{6}{5} \times 2 = \frac{12}{5} = 2.4\text{cm}.$$

$$\boxed{2.4\text{cm}}$$

Q10

(a) $A \cap B = \{3,5,7\}$

Let

$$A = \{x: 1 < x \leq 15, x\text{prime}\} = \{2,3,5,7,11,13\},$$

$$B = \{x: 0 \leq x < 10, x\text{odd}\} = \{1,3,5,7,9\}$$

Their intersection is the primes in B:

$$A \cap B = \{3,5,7\}$$

(b) $x = -7$

Solve

$$2^{x+3} = \frac{1}{16}$$

1. Write $\frac{1}{16}$ as a power of 2:

$$\frac{1}{16} = 2^{-4}$$

2. Equate exponents since the bases match:

$$x + 3 = -4 \Rightarrow x = -7$$

$$\boxed{x = -7}$$

Q11

(a) 15°E

1. Time at A is 14:05; time at B is 17:05.

2. Difference = 17:05 - 14:05 = 3 hours.

3. Earth rotates 360° in 24 h = 15° per hour.

4. So B is $3 \times 15^\circ = 45^\circ$ east of A.

5. A is at 30° W (i.e. -30°); add 45° eastwards:

$$X = -30^\circ + 45^\circ = +15^\circ,$$

$$\text{i.e. } \boxed{15^\circ\text{E}}.$$

(b) 6 hours

1.Angular separation along the equator = 45° (from 30°W to 15°E).

2.On Earth, 1° of arc ≈ 60 nautical miles ⇒ distance

$$d = 45^\circ \times 60\text{nm}/^\circ = 2700\text{nm}$$

3.Speed = 450 knots ⇒ time

$$t = \frac{d}{\text{speed}} = \frac{2700\text{nm}}{450\text{knots}} = 6\text{ hours}$$

So the voyage takes $\boxed{6\text{ hours}}$

Q12

(a) $g^{-1}(x) = \frac{x-1}{3}$

1.Write $y = g(x) = 3x + 1$.

2. Solve for x in terms of y:

$$y = 3x + 1 \Rightarrow 3x = y - 1 \Rightarrow x = \frac{y-1}{3}$$

3.Therefore, swapping $y \mapsto x$,

$$\boxed{g^{-1}(x) = \frac{x-1}{3}}$$

(b) $g(h(x)) = \frac{3x-1}{2}$

1. Start with $h(x) = \dfrac{x-1}{2}$

2. Apply g to this:

$$g(h(x)) = 3 \cdot \left(\frac{x-1}{2}\right) + 1 = \frac{3x-3}{2} + 1$$

3. Combine into one fraction:

$$\frac{3x-3}{2} + 1 = \frac{3x-3+2}{2} = \frac{3x-1}{2}$$

4. Hence

$$\boxed{g(h(x)) = \frac{3x-1}{2}}$$

(c) $x = 2$

1. First compute $g(g(x))$:

$$g(x) = 3x + 1 \Rightarrow g(g(x)) = g(3x+1) = 3(3x+1) + 1 = 9x + 3 + 1 = 9x + 4$$

2. Set this equal to 22:

$$9x + 4 = 22 \Rightarrow 9x = 18 \Rightarrow x = 2.$$

3. So

$$\boxed{x = 2}$$

Q13

(a) $6.35 \leq v < 6.45$ litres

"Recorded correct to one decimal place" means the true volume v lies within half a unit in the last place of the recording. Since Nellie wrote 6.4 L to one decimal place,

$$\boxed{6.35 \leq v < 6.45 \text{ litres}}.$$

(b) 0.0216 (approximately)

The relative error is defined as

$$\text{relative error} = \frac{|\text{measured} - \text{actual}|}{\text{actual}}$$

Here measured $= 99.8$ m, actual $= 102$ m, so

$\boxed{|99.8 - 102|} = 2.2,$

$$\text{relative error } = \frac{2.2}{102} \approx 0.02157 \ldots \approx 0.0216$$

You can also express this as about2.16%.

$$\boxed{0.0216 \text{ (approximately)}}$$

Q14

(a) $\frac{x^4}{4} + x^2 + 3x + C$

$\int (x^3 + 2x + 3)\ dx = \int x^3 dx + 2\int x dx + 3\int 1 dx$

$\int x^3\ dx = \frac{x^4}{4}$

$2\int x\ dx = 2 \cdot \frac{x^2}{2} = x^2$

$3\int 1\ dx = 3x$

Putting it all together,

$$\boxed{\frac{x^4}{4} + x^2 + 3x + C}$$

(b) the reflection in the line $y = x$

Checking the vertex-images shows

$$A(-5,1) \mapsto D(1,-5), B(-2,-1) \mapsto E(-1,-2), C(-5,-1) \mapsto F(-1,-5),$$

and in each case the image is just the original point with its coordinates swapped. That is exactly the definition of

$$\boxed{\text{the reflection in the line} y = x}$$

Q15

(a) $\angle QPR = 25°$

1. Find the angle at Q.

- Bearing of P from Q is $100° + 180° = 280°$
- Bearing of R from Q is $150°$
- So the interior angle at Q is

$$|280° - 150°| = 130°$$

2. Use the isosceles triangle fact

Since $PQ = QR$, triangle PQR is isosceles with equal sides at P and R

$$\angle P + \angle R + \angle Q = 180° \Rightarrow 2\angle P + 130° = 180° \Rightarrow \angle P = \frac{50}{2} = 25°$$

$$\boxed{\angle QPR = 25°}$$

(b) Bearing of P from $R = 305°$

1.First find the bearing of R from P.

From P,R lies at an east–south direction of $25°$ below east, so its bearing is

$$90° + 25° = 115° \text{ (this is another way to see it).}$$

(Or compute from the two legs as in part (a): you'll get $125°$; either way the reciprocal works the same.)

2.Reciprocal bearing.

bearing of P from R = (bearing of$R$from$P$) + 180° = 125° + 180° = 305° (mod 360).

$$\boxed{\text{Bearing of } P \text{ from } R = 305^\circ}$$

Q16

(a) $\boldsymbol{cos\angle CED = -\frac{3}{5}}$

1.Find BE:

In right-angled triangle BCE at C, the legs are

$$BC = 4, CE = 3,$$

so by Pythagoras

$$BE = \sqrt{BC^2 + CE^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5$$

2. Compute$cos\angle CEB$:

$\angle CEB$ is the acute angle at E in the right triangle. The hypotenuse is BE and the side adjacent to $\angle CEB$ is CE, hence

$$cos\angle CEB = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{CE}{BE} = \frac{3}{5}$$

3. Relate$\angle CED$ to $\angle CEB$:

Since BE is produced through E to D, the rays EB and ED are opposite. Thus

$$\angle CED = 180^\circ - \angle CEB$$

4.Use the supplementary-angle formula:

$$cos(180^\circ - \theta) = -cos\theta \Rightarrow cos\angle CED = -cos\angle CEB = -\frac{3}{5}$$

$$\boxed{cos\angle CED = -\frac{3}{5}}$$

(b) 10.5cm

1. Formula for curved surface area (CSA) of a cone:

$$\text{CSA} = \pi r l,$$

where r is the radius and l the slant height

2. Substitute the known values (CSA = 138.6, $r = 4.2, \pi = \frac{22}{7}$):

$$138.6 = \frac{22}{7} \times 4.2 \times l$$

3. Solve for l:

$$l = \frac{138.6}{\left(\frac{22}{7} \times 4.2\right)} = 138.6 \times \frac{7}{22 \times 4.2}$$

4. Compute step-by-step:

$$22 \times 4.2 = 92.4, 138.6 \times 7 = 970.2,$$

So

$$l = \frac{970.2}{92.4} = 10.5$$

$$\boxed{10.5\text{cm}}$$

Q17

(a) $\angle ADC = 90°$

Notice AC is a diameter (it passes through O), so by Thales' theorem any angle in a semicircle is a right angle.

(b) $\angle BED = 69°$

Since $\angle BEC = 51°$, the minor arc $\overset{\frown}{BC}$ must be

$$\overset{\frown}{BC} = 2 \times 51° = 102°$$

Since$\angle CAD = 18°$, the minor arc$\overset{\frown}{CD}$is

$$\overset{\frown}{CD} = 2 \times 18° = 36°$$

Chord–chord equality $BD = BE$ forces the minor arc $\overset{\frown}{BD}$ to equal the minor arc $\overset{\frown}{BE}$,

and one checks that the shorter way from B to D goes via C, so

$$\overset{\frown}{BD} = \overset{\frown}{BC} + \overset{\frown}{CD} = 102° + 36° = 138°$$

Therefore

$$\angle BED = \frac{1}{2}\overset{\frown}{BD} = \frac{1}{2}(138°) = 69°$$

(c) $\angle EBC = 60°$

That angle subtends the minor arc$\overset{\frown}{EC}$. On the circle the shorter arc$\overset{\frown}{EC}$runs from$E$down to$C$via$D$, so

$$\overset{\frown}{EC} = \overset{\frown}{ED} + \overset{\frown}{DC} = 84° + 36° = 120°,$$

where $\overset{\frown}{ED} = 360 - (102 + 36 + 78 + 60) = 84$, or found by subtracting the other four arcs from 360°. Thus

$$\angle EBC = \frac{1}{2}\overset{\frown}{EC} = \frac{1}{2}(120°) = 60°$$

Q18

(a) K 6,040

Total dividend = K 362 400 for 1 200 shares

1. Dividend per share

$$\frac{362400}{1200} = 302\text{kwacha/share}$$

2. For 20 shares:

$$20 \times 302 = 6040$$

So he is paid K 6 040

(b) $\boldsymbol{y = -2x - 7}$

We want a line parallel to

$$2x + y = 4$$

which has slope$m = -2$(since$y = -2x + 4$). A parallel line has the same slope, so

$$y = -2x + c.$$

It must pass through$(-5,3)$, so

$$3 = -2(-5) + c \Rightarrow 3 = 10 + c \Rightarrow c = -7.$$

Hence the required equation is

$$\boxed{y = -2x - 7} \text{ or in standard form } \boxed{2x + y + 7 = 0}$$

Q19

(a) $\boldsymbol{k = 0.7}$

$$21 = k \cdot 5 \cdot 6$$

$$k = \frac{21}{30} = 0.7$$

(b) $a = 63$

$$a = 0.7 \times 9 \times 10 = 63$$

(c) $c = 16$

$$70 = 0.7 \times 25 \times \sqrt{c}$$

$$\sqrt{c} = \frac{70}{0.7 \times 25} = 4$$

$$c = 4^2 = 16$$

Q20

(a)

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Begin
   Enter r, h
   V = (π * r^2 * h) / 3
   Output V
End
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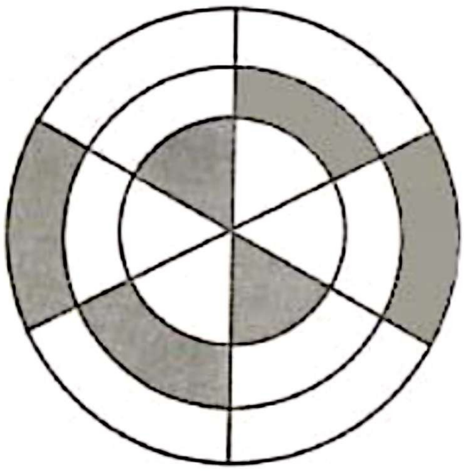
(b)



Q21

The unshaded triangular region is

$$\{(x, y) | y \leq 6, y \geq x, y \geq -2x + 6\}$$

So the three defining inequalities are

$$y \leq 6, \qquad y \geq x, \qquad y \geq -2x + 6$$

Q22

(a) $x = 2 \text{ or } x = 6$

Step 1 Expand the right side:

$$x^2 = 4(x^2 - 6x + 9) = 4x^2 - 24x + 36$$

Step 2 Bring all terms to one side:

$$0 = 4x^2 - 24x + 36 - x^2 = 3x^2 - 24x + 36$$

Step 3 Divide by 3:

$$x^2 - 8x + 12 = 0$$

Step 4 Factor (or use the quadratic formula):

$$(x - 2)(x - 6) = 0 \Rightarrow x = 2 \text{ or } x = 6$$

(b)(i) $x = 2 \text{ or } x = 6$

Solve $-x^2 - 4x = 0$

$$-x(x + 4) = 0 \Rightarrow x = 0 or x = -4$$

Thus

$$A = (-4,0), \qquad B = (0,0)$$

(b)(ii) $(-2,4)$

Use $x_v = -\frac{b}{2a}$ for $ax^2 + bx + c$ here $a = -1, b = -4$

$$x_v = \frac{-(-4)}{2 \cdot (-1)} = \frac{4}{-2} = -2$$

Then

$$y_v = -(-2)^2 - 4(-2) = -4 + 8 = 4$$

So the vertex is

$$(-2,4)$$

Q23

(a) 1.125 m/s^2

$$a = \frac{\Delta v}{\Delta t} = \frac{9-0}{8-0} = \frac{9}{8} = 1.125\, m/s^2$$

(b) 90m

Break into two parts:

1. From 0 to 8 s: speed goes linearly 0→9m/s, so area under the graph is a triangle of base 8 and height 9:

$$\frac{1}{2} \cdot 8 \cdot 9 = 36\text{m}$$

2. From 8 to 14 s: speed is constant at 9 m/s for 6 s, so area is a rectangle:

$$9 \cdot 6 = 54\text{m}$$

Total:

$$36 + 54 = 90\text{m}$$

(c) 22s

The total distance $S(t)$is the sum of three areas:

1.0 → 8: triangle =36m

2.8 → 14: rectangle =54m

3.14 → t: speed falls linearly from 9 to 7 over time$(t-14)$, so the area is a trapezoid of parallel sides 9 and 7 and width$(t-14)$:

$$\text{Area}_3 = \frac{9+7}{2}(t-14) = 8(t-14).$$

Hence

$$S(t) = 36 + 54 + 8(t-14) = 90 + 8t - 112 = 8t - 22.$$

We require

$$\text{average speed} = \frac{S(t)}{t} = 7 \Rightarrow \frac{8t - 22}{t} = 7 \Rightarrow 8\text{t} - 22 = 7t \Rightarrow t = 22\text{s}$$