Q1.

To evaluate $\left(\frac{3}{2}\right)^{-2}$

1. Invert the fraction and change the exponent to positive:

$$\left(\frac{2}{3}\right)^{-2} = \frac{2^2}{3^2} = \frac{4}{9}$$

2. Square the numerator and the denominator:

$$\left(\frac{2}{3}\right)^{-2} = \frac{2^2}{3^2} = \frac{4}{9}$$

So, $\left(\frac{3}{2}\right)^{-2} = \frac{4}{9}$

Q2.

To simplify $3(4x - 5) + 2$:

1. Distribute the 3 inside the parentheses:

$$3 \cdot 4x + 3 \cdot (-5) + 2$$

2. Perform the multiplications:

$$12x - 15 + 2$$

Combine like terms:

$$12x - 13$$

So, the simplified form is:

$$12x - 13$$

Q4.

To solve the equation $4y^2 - 8y = 0$:

1. Factor out the greatest common factor, $4y$:

$$4y(y - 2) = 0$$

2. Set each factor equal to zero:

$$4y = 0 \text{ or } y - 2 = 0$$

3. Solve each equation:

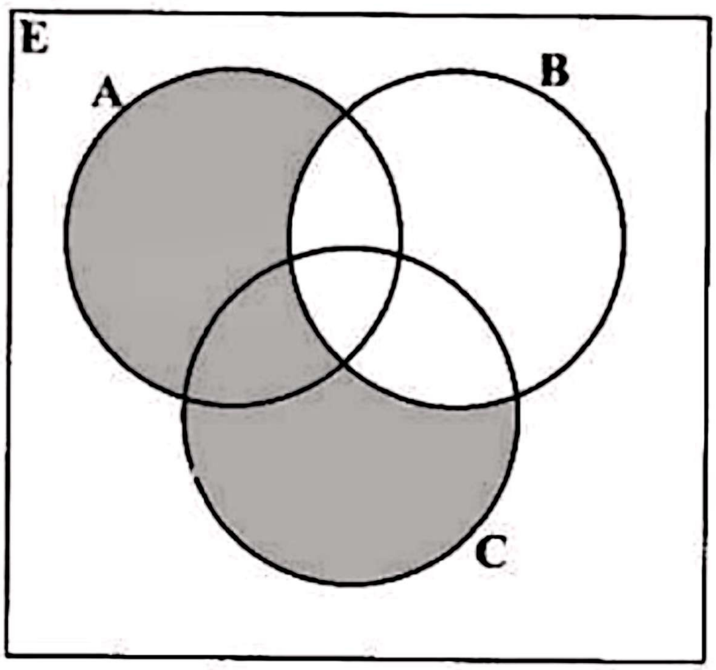
$$y = 0 \text{ or } y = 2$$

So, the solutions are:

$$y = 0 \text{ and } y = 2$$

Q5.

Here is the Venn diagram showing the shaded region representing $(A \cup B') \cap C$. The shaded area includes parts of circle A and parts of circle C that are outside circle B.



Q6.

To find out how much dividend the businessman received:

1. Dividend per share: $K2.75$

2. Number of shares: 560

Calculate the total dividend:

$$Total\ Dividend = Dividend\ per\ Share \times Number\ of\ Shares$$

$$Total\ Dividend = 2.75 \times 560$$

Perform the multiplication:

$$2.75 \times 560 = 1540$$

So, the businessman received a total dividend of $K1540$.

Q7.

(a) Common Difference, d:

The common difference $d$is found by subtracting the first term from the second term:

$$d = -1 - 2 = -3$$

(b) Formula for the nth Term:

1. The formula for the nnth term of an arithmetic progression is given by:

$$an = a + (n - 1)d$$

where a is the first term and d is the common difference.

2. For this sequence, the first term $a = 2$ and the common difference $d = -3$.

Substitute aa and d into the formula:

$$a_n = 2 + (n - 1)(-3)$$

4. Simplify the expression:

$$a_n = 2 - 3(n - 1)$$

$$a_n = 2 - 3n + 3$$

$$a_n = 5 - 3n$$

So, the formula for the nth term is:

$a_n = 5 - 3n$

Q8.

(a)

1. The probability that the boy will not be late is the complement of the probability that he will be late.

2. The complement rule states that the probability of an event not happening is 11 minus the probability of the event happening.

So, the probability that he will not be late for school is:

$1 - x$

(b)

1. The vector $\overrightarrow{RS}$ represents the displacement from R to S.

2. The coordinates of S can be found by adding the vector $\overrightarrow{RS}$ to the coordinates of R.

Let the coordinates of R be (x,y).

Using the vector addition:

$$\overrightarrow{RS} = \begin{pmatrix} S_x - R_x \\ S_y - R_y \end{pmatrix}$$

Given:

$$\begin{pmatrix} -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 - x \\ 2 - y \end{pmatrix}$$

Set up the equations:

$$-4 = 1 - x$$

$$5 = 2 - y$$

Solve for x:

$$1 - x = -4$$

$$-x = -4 - 1$$

$$-x = -5$$

$$x = 5$$

Solve for y:

$$2 - y = 5$$

$$-y = 5 - 2$$

$$-y = 3$$

$$y = -3$$

So, the coordinates of point R are:

$(5, -3)$

Q9

(a)

The transpose of matrix M is obtained by swapping rows with columns.

$$M^T = \begin{pmatrix} 1 & 5 \\ 2 & 7 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ 5 & 7 \end{pmatrix}$$

(b)

To find the product NM, multiply each element of the rows of N by the corresponding elements of the columns of M, and sum the products.

$$NM = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 2 & 7 \end{pmatrix}$$

Calculate the elements of NM:

1. First row, first column:

$$(0 \cdot 1) + (1 \cdot 2) = 0 + 2 = 2$$

2. First row, second column:

$$(0 \cdot 5) + (1 \cdot 7) = 0 + 7 = 7$$

3. Second row, first column:

$$(2 \cdot 1) + (0 \cdot 2) = 2 + 0 = 2$$

4. Second row, second column:

$$(2 \cdot 5) + (0 \cdot 7) = 10 + 0 = 10$$

So, the product NM is:

$$NM = \begin{pmatrix} 2 & 7 \\ 2 & 10 \end{pmatrix}$$

Q10

(a)

1. The intersection of two sets X and Y is the set of elements that are common to both X and Y.

2. List the elements of both sets:

- Elements of X: 1,5,9
- Elements of Y: 3,9,11

3. Find the common elements:

- The common element between X and Y is 9.

So, the intersection $X \cap Y$ is:

$$X \cap Y = 9$$

(b)

Transformation:

- The transformation is a reflection across the line $y = 2$.

Verification:

- The vertical distance from the vertices of triangle AA to the line $y = 2$ matches the vertical distance from the line $y = 2$ to the corresponding vertices of triangle BB.

Q11

(a)

Given:

- Local time at A is 10:00 hours.
- Local time at B is 13:00 hours.

1. Time difference between A and B:

$$Time\ difference = 13{:}00 - 10{:}00 = 3\ hours$$

2. Each hour corresponds to 15 degrees of longitude (since the Earth rotates 360 degrees in 24 hours):

$$Difference\ in\ longitudes = 3\ hours \times 15\ degrees/hour = 45\ degrees$$

So, the difference in longitudes between A and B is 45 degrees.

(b)

Given:

- Speed of the plane = 600 knots.
- Points B and C are on different latitudes:
  - B: 60°S
  - C: 30°N

1. Calculate the total distance traveled by the plane in nautical miles:

- The Earth is divided into 180 degrees from pole to pole (90 degrees from the equator to each pole).
- The total latitude difference from 60°S to 30°N:

$$Total\ latitude\ difference = 60 + 30 = 90\ degrees$$

2. Each degree of latitude is approximately 60 nautical miles:

$$Total\ distance = 90\ degrees \times 60\ nautical\ miles/degree = 5400\ nautical\ miles$$

3. Calculate the time taken:

$$Time = \frac{Distance}{Speed} = \frac{5400\ nautical\ miles}{600\ knots} = 9\ hours$$

So, the plane took 9 hours to fly from B to C.

Q12

(a)

The formula for the gradient mm between two points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Given:

$A(4, -6) and B(2,4)$

Substitute the coordinates into the formula:

$$m = \frac{4 - (-6)}{2 - 4}$$

Simplify the numerator and the denominator:

$$m = \frac{4 - (-6)}{2 - 4} = \frac{10}{-2} = -5$$

So, the gradient of the line passing through the points A and B is:

$$m = -5$$

(b)

1. Divide both sides by 2:

$$x^3 = \frac{16}{2}$$

$$x^3 = 8$$

2. Take the cube root of both sides:

$$x = \sqrt[3]{8}$$

$$x = 2$$

So, the solution to the equation $2x^3 = 16$ is:

$$x = 2$$

Q13

(a) *Calculate angle* $\angle ACB$

1. Since AB and AC are tangents to the circle from point A, they are equal in length.

2. Hence, $\triangle ABC$ is an isosceles triangle.

3. Given $\angle BAC$=54∘.

Since $\triangle ABC$ is isosceles, the base angles are equal. Let $\angle ACB = \angle ABC = x$.

The sum of angles in $\triangle ABC$ is 180°:

$$54 \circ + x + x = 180°$$

$$54° + 2x = 180°$$

$$2x = 180° - 54°$$

$$2x = 126°$$

$$x = 63°$$

So, $\angle ACB = 63°$.

(b) Calculate angle $\angle CBD$

1. AB and BE are straight lines.

2. $\angle CBD$ is the exterior angle for $\triangle ABC$.

The exterior angle $\angle CBD$ is equal to the sum of the two opposite interior angles:

$$\angle CBD = \angle BAC + \angle ACB$$

$$\angle CBD = 54° + 63°$$

$$\angle CBD = 117°$$

(c) Calculate angle $\angle CDB$

1. AC is tangent to the circle at C, and BE is tangent to the circle at B.

2. The line DE is a straight line through points D and E.

3. $\angle CDB$ is the supplementary angle of $\angle CBD$.

The sum of angles on a straight line is 180°:

$$\angle CDB = 180° - \angle CBD$$

$$\angle CDB = 180° - 117°$$

$$\angle CDB = 63°$$

Q14

(a)

When a value is given to 1 decimal place, the actual value could be up to 0.05g more or less than the given value.

- The upper limit is $702.1 + 0.05 = 702.15g$.
- The lower limit is $702.1 - 0.05 = 702.05g$.

So, the lower limit is:

$$702.05g$$

(b)

The relative error is given by the absolute error divided by the measured value. The absolute error is half the precision of the measurement.

- Absolute error: $\pm 0.05 \pm 0.05$g.
- Measured value: 702.1702.1g.

Relative error:

$$Relative\ error = \frac{Absolute\ error}{Measured\ value}$$

$$Relative\ error = \frac{0.05}{702.1}$$

Calculate the relative error:

$$Relative\ error \approx 7.12 \times 10^{-5} (to\ 5\ significant\ figures)$$

So, the relative error of the mass of the loaf of bread is approximately:

$$7.12 \times 10^{-5}$$

Q15

(a)

Given:

- Volume of smaller solid $P = 80\ cm^3$

- Volume of larger solid $Q = 270\,cm^3$
- Height of smaller $solid\ = 8\,cm$

To find:

- Height of the larger solid.

Since the solids are similar, the ratio of their volumes is the cube of the ratio of their corresponding linear dimensions (heights).

Let the height of the larger solid be h.

The volume ratio is given by:

$$\left(\frac{Height\ of\ larger\ solid}{Height\ of\ smaller\ solid}\right)^3 = \frac{Volume\ of\ larger\ solid}{Volume\ of\ a\ smaller\ solid}$$

Substitute the given values:

$$\left(\frac{h}{8}\right)^3 = \frac{270}{80}$$

Simplify the ratio on the right:

$$\left(\frac{h}{8}\right)^3 = \frac{27}{8}$$

Taking the cube root of both sides:

$$\left(\frac{h}{8}\right) = \sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$$

Solve for h:

$$h = 8 \times \frac{3}{2} = 8 \times 1.5 = 12cm$$

(b)

1. Identify the Triangle:

- Given triangle EFG is a right-angled triangle at F.
- $FG = 8\,cm$
- $EG = 10\,cm$

2. Find EF Using Pythagoras' Theorem:

$EG^2 = EF^2 + FG^2$

$10^2 = EF^2 + 8^2$

$100 = EF^2 + 64$

$EF^2 = 100 - 64$

$EF^2 = 36$

$EF = 6cm$

(c) Find $sin\angle EGH$ $sin\angle EGH$:

- $\angle EGH$ is the same as $\angle EGF$ since FGH is a straight line.
- $sin\angle EGF = \frac{Opposite}{Hypotenuse}$
- Opposite side to $\angle EGF$ is FG.
- Hypotenuse is EG.

$$sin\angle EGH = \frac{FG}{EG} = \frac{8}{10} = 0.8$$

$So, sin\angle EGH = 0.8.$

Q16

(a)

Given $y = 2y = 2, x = 4x = 4$, and $z = 24$:

$$2 = k\frac{4^2}{24}$$

$$2 = k\frac{16}{24}$$

$$2 = k\frac{2}{3}$$

$$2 = 2 \times \frac{2}{3}$$

$$k = 3$$

(b)

Using the equation $y = k\frac{x^2}{z}$ $with\ k = 3$

$$y = 3\frac{9^2}{27}$$

$$y = 3\frac{81}{27}$$

$$y = 3 \times 3$$

$$y = 9$$

(c)

Using the equation $y = k\frac{x^2}{z}$ $with\ k = 3$

$$8 = 3\frac{x^2}{6}$$

$$8 = \frac{3x^2}{6}$$

$$8 = \frac{x^2}{2}$$

$$x^2 = 16$$

$$x = \sqrt{16}\ \ or\ x = -\sqrt{16}$$

$$x = 4\ \ or\ \ x = -4$$

So, the values of x are:

$$x = 4\ \ or\ \ x = -4$$

Q17

(a)

A regular triangular prism has several types of symmetry:

1. **Rotational Symmetry:**

- The prism can be rotated around its longitudinal axis (the axis running through the centers of the triangular bases) by 120∘120∘ or 240∘240∘ and still look the same. This gives it rotational symmetry of order 3.

2. **Reflectional Symmetry:**
    - The prism has three planes of symmetry:
        - One plane of symmetry passes through the midpoints of the edges connecting the two triangular bases, dividing the prism into two mirror-image halves.
        - The other two planes of symmetry pass through the axis of the prism and the vertices of the triangular bases, dividing the prism into two mirror-image halves along different planes.
3. **Translational Symmetry:**
    - The prism has translational symmetry along the length of the prism. If it is shifted along its length, it will remain unchanged.

(b)

Start

Enter r, s

A = 3.14159 * r * s

Output A

Stop

Q18

(a)

Given:

- $A(2,1)A(2,1)$
- $B(-6,5)B(-6,5)$
- B is the midpoint of AC.

To find the coordinates of C, we use the midpoint formula:

$$B\left(\frac{x_{1+}x_2}{2}, \frac{y_2 + y_2}{2}\right)$$

where $(x1, y1)$ are the coordinates of A and $(x2, y2)$ are the coordinates of C.

Given:

$$B(-6{,}5) = \left(\frac{2 + x_2}{2} + \frac{1 + y_2}{2}\right)$$

Solving for x_2:

$$-6 = \frac{2 + x_2}{2}$$

Multiply both sides by 2:

$$-12 = 2 + x_2$$

Subtract 2 from both sides:

$$x_2 = -14$$

Solving for y_2:

$$5 = \frac{1 + x_2}{2}$$

Multiply both sides by 2:

$$10 = 1 + y_2$$

Subtract 1 from both sides:

$$y_2 = 9$$

Coordinates of C:

$$C(-14{,}9)$$

So, the coordinates of C are $(-14{,}9)$.

(b)

Formula for the area of a sector:

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

Given values:

- $Angle\ \theta = 72°$

- Radius $r = 14cm$
- $\pi = \frac{22}{7}$

Substitute the values into the formula:

$$\text{Area of sector} = \frac{72°}{360°} \times \frac{22}{7} \times 14^2$$

Simplify the fraction:

$$\frac{72°}{360°} = \frac{1}{5}$$

Calculate the area:

$$\text{Areao f sector} = \frac{1}{5} \times \frac{22}{7} \times 196$$

$$= \frac{1}{5} \times 22 \times \frac{196}{7}$$

$$= \frac{1}{5} \times 22 \times 28$$

$$= 123.2cm^2$$

The area of the sector AOB is $123.2cm^2$

Q19

(a)

$Start\ with\ y = f(x)$

$$y = f(x) = 3x + 1$$

$Solve\ for\ x$

$$y - 1 = 3x$$

$$x = \frac{y - 1}{3}$$

$Replace\ y\ with\ x\ to\ find\ f^{-1}(x)$:

$$f^{-1}(x) = \frac{x - 1}{3}$$

(b)

$$f^{-1}(x) = \frac{x-1}{3}$$

$$f^{-1}(-5) = \frac{-5-1}{3}$$

$$f^{-1}(-5) = \frac{-6}{3}$$

$$f^{-1}(-5) = -2$$

(c)

1. Use the given functions $f(x) = 3x + 1$ *and* $g(x) = 4x - 1$

2. Substitute $g(x)$ into $f(x)$:

$$f(g(x)) = f(4x - 1)$$

3. Apply f *to* $4x - 1$:

$$f(4x - 1) = 3(4x - 1) + 1$$

$$3(4x - 1) = 12x - 3 + 1$$

$$f(g(x)) = 12x - 2$$

Q20

Given:

- $\angle BAC = 46°$
- $AC = BC$
- $B \text{ is due east of } A$

(a)

1. Since $AC = BC$, $\triangle ABC$ is an isosceles triangle.

2. The angles opposite the equal sides are equal. Thus, $\angle ACB = \angle ABC$.

3. The sum of angles in a triangle is 180°. Therefore:

$$\angle BAC + \angle ACB + \angle ABC = 180°$$

$$46° + 2\angle ACB = 180°$$

$$2\angle ACB = 134°$$

$$\angle ACB = 67°$$

Since the bearing of A from C is measured clockwise from north: $360° - 67° = 293°$

The bearing of A from C is $293°$.

(b)

1. The bearing of C from B is the angle measured clockwise from north.
2. Since B is due east of A, the north direction at B is the same as at A.
3. The bearing of C from B is equal to the angle $\angle ACB$ plus $90°$ (since B is due east of A):

$$\angle ACB + 90° = 67° + 90° = 157°$$

The bearing of C from B is $157°$.

Q21

(a)

Vertical lines:

- The line at $x = -2$: Since R is to the right of this line, the inequality is $x \geq -2$.
- The line at $x = 4$: Since R is to the left of this line, the inequality is $x \leq 4$.

Horizontal line:

- The line at $y = 2$: Since R is above this line, the inequality is $y \geq 2$.

Diagonal line:

The line passes through the points (0,6) and (8,0). To find the equation of this line:

$$slope = \frac{0-6}{8-0} = \frac{6}{8} = -\frac{3}{4}$$

Using the point-slope form of the line equation $y - y_1 = m(x - x_1)$, where (0,6) is a point on the line:

$$y - 6 = -\frac{3}{4}x$$

Simplifying to slope-intercept form:

$$y = -\frac{3}{4}x + 6$$

Since R is below this line, the inequality is

$$y \leq -\frac{3}{4}x + 6$$

Combining these, the inequalities that define the unshaded region R are:

$$\begin{cases} x \geq -2 \\ x \leq 4 \\ y \geq 2 \\ y \leq -\frac{3}{4}x + 6 \end{cases}$$

Q22

(a)

Differentiate $2x^2$:

$$\frac{d}{dx}(2x^2) = 2 \cdot 2x = 4x$$

Differentiate $-4x$:

$$\frac{d}{dx}(-4x) = -4$$

Differentiate the constant 3:

$$\frac{d}{dx}(3) = 0$$

Combine the results:

$$\frac{dy}{dx} = 4x - 4$$

Thus, $\frac{dy}{dx} = 4x - 4$

(b) (i)

These points are where the graph intersects the x-axis. To find the x-intercepts, set $y = 0$:

$$x^2 - 2x - 8 = 0$$

Solve the quadratic equation:

$$x^2 - 2x - 8 = 0$$

1. Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2}$$

$$x = \frac{2 \pm 6}{2}$$

Solve for the two values of x:

$$x = \frac{2+6}{2} = \frac{8}{2} = 4$$

$$x = \frac{2-6}{2} = \frac{-4}{2} = -2$$

Thus, the coordinates of A and B are:

$A(-2,0)$

$B(4,0)$

(ii)

1. For the equation $y = x^2 - 2x - 8$

$a = 1,\ b = -2$

$$x = -\frac{-2}{2 \cdot 1} = -\frac{2}{2} = 1$$

2. Find the y-coordinate by substituting $x = 1$ back into the equation:

$y = 1^2 - 2 \cdot 1 - 8$

$y = 1 - 2 - 8$

$y = -9$

Thus, the coordinates of the minimum point are:

$(1, -9)$

Q23

(a)

The object decelerates from 10 m/s to 0 m/s in 5 seconds.

$$Retardation = \frac{\Delta v}{\Delta t}$$

$$= \frac{0 - 10\text{m/s}}{5\text{s}}$$

$$= -2\text{m}/s^2$$

(b)

The first 10 seconds include:

- Accelerating for 2 seconds
- Constant speed for 8 seconds

1. Distance during acceleration (first 2 seconds)

$$\text{Distance} = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times 2\text{s} \times 10\text{m/s}$$

$$= 10\text{m}$$

2. Distance during constant speed (next 8 seconds)

Distance during constant speed: $10\text{m/s} \times 8\text{s} = 80\text{m}$

Total distance in first 10 seconds

Total distance = $10\text{m} + 80\text{m} = 90\text{m}$

(c)

1. Distance during deceleration (last 5 seconds)

$$= 10 \times 5 + \frac{1}{2} \times (-2) \times 5^2$$

$$= 50 + \frac{1}{2} \times (-2) \times 25$$

$= 50 - 25$

$= 25m$

2. Total distance traveled

Total distance = 10m +80m +25m =115m

3. Total time

Total time: = 2s +8s +5s

=15s

4. Average speed

$$\text{Average speed} = \frac{Total\ Distance}{Total\ time}$$

$$= \frac{155\text{m}}{15\text{s}}$$

$$= \frac{155}{15}$$

$$\approx 7.67\text{m/s}$$